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Supporting Inquiry Skills through Mathematical Competition B-Day

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Abstract. The article describes the possibilities of development of inquiry skills of secondary school students during preparing and participating in mathematical competition B-Day. During the competition teams of three or four secondary school students work on open-ended tasks that require students' investigation and original approach to the problem. The competition is not only about finding the winner, but it is suitable for identifying the level of students' inquiry skills.

Keywords: Inquiry based learning, mathematics competition, inquiry processes

Classification: D54

1 Introduction

Secondary mathematics education continuously goes through several organisational changes that influence for example the reduction of mathematical content, the implementation of new teaching methods. The changes should be understood within the aims of mathematics education. They can be simplify to: mastery in basic skills in mathematical strands, to know how to apply this knowledge and to be able to approach new open problems in mathematics inquiry. All of them are important, but especially the third aim is crucial for many technical or applied mathematical professions. The first and partly the second aim can be evaluated during the standardised test. We agreed with the opinion as mentioned in [1] that it is hard to evaluate the inquiry skills within the two-hour standardised test.

In our contribution, we describe the pilot study of a successful mathematics competition B-DAY in Slovak settings. We consider the competition as a good example where pupils can experience mathematical investigation and use several inquiry processes. In the competition, pupils are supposed to work on complex tasks where they need to investigate unknown mathematical areas to find the solution. On the other side, the results of this kind of competition offer us good environment for monitoring pupils' inquiry processes.

2 Background of the competition

The Mathematics B-DAY is a mathematics competition for teams of secondary-school pupils at the age of 15-19. Although the competition was organized in Slovakia in the school year 2011/2012 for the first time, in the Netherlands, Germany and Belgium it has its tradition for several years. The main organiser of the competition is the Freudenthal Institute for Science and Mathematics Education of Utrecht University in Netherlands. In Netherlands, "Mathematics B is the mathematics needed for technical studies and studies in science and mathematics at university level, its core component is calculus. The assignment is related to the Mathematics A-lympiad. As in the A-lympiad the real world character of the task is essential, for the Mathematics B-day the emphasis is on mathematical problem solving, conjecturing and proving."

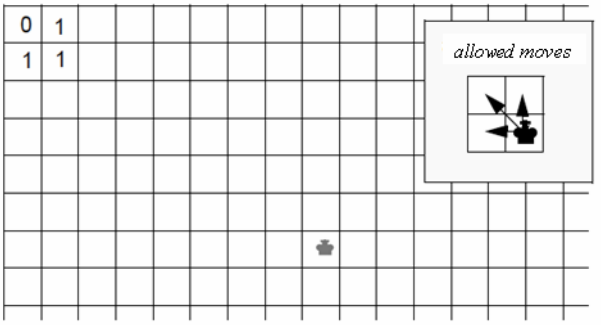

Figure 1: Possible moves of the king

3 B-DAY in Slovakia

In school year 2011/2012, the Mathematics B-DAY competition was introduced for the first time in Slovakia within the international project 7FP PRIMAS (www.primas-project.eu). One of the main aims of the project is to create the environment and realize the activities suitable for the implementation of inquiry-based learning in mathematics education. The competition was organized by partners of the project PRIMAS from the Department of Mathematics at Faculty of Natural Sciences, Constantine the Philosopher University in Nitra in collaboration with the partners of the project from the Freudenthal Institute of Utrecht University, Netherlands.

4 Assignment from 2011

The theme of the assignment and the context of the problems differ every year. In 2011, the title of the assignment for the Mathematics B-DAY competition was "The final move". The assignment was formally divided into four parts. The first part presented four combinatorial games which were intended to familiarize the pupils with looking for a winning strategy. The second part contained a theoretical background applicable to the games from the first part of the assignment. The third part included a detailed analysis of one game. The last part was a final assignment which was the main part of pupils' mathematical inquiry.The assignment respects the principle of lower level of knowledge to enter but quite high mathematical outcome of the possible results.

As an example of the scaffolding character of the tasks we present the task about the problems on the chessboard. This problem is presented in every part of the assignment. The king can move only up, left or diagonal up - left (See Figure 1). What pupils had to do was to indicate precisely which starting points will allow the first player to win, no matter how well the opponent plays. Other starting points mean the second player wins, no matter how well the first player plays. After playing the game pupils get the understanding of the solution.

- Try to find the winning and losing starting points.

In the second part, pupils were asked to deepen their understanding of the concept as well as to develop the appropriate terminology that was necessary for the final assignment. Pupils should focus more on the exploring, formulating hypothesis

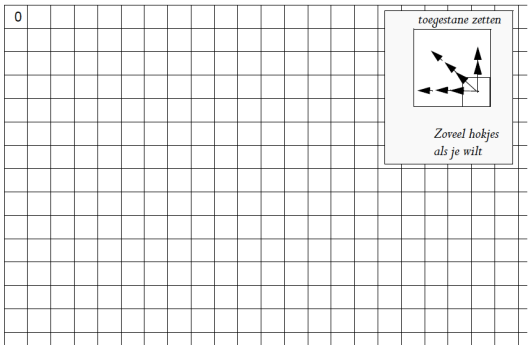

Figure 2: Possible moves of the queen

and finding conjectures. The tasks go different directions as listed in the first part but have common ways of solution(See Figure 2). The task follows as: :

1. So why do you have to put a 0 in square (0, 2)?
2. Now you can continue with square (1, 2). Why must that be a 1?
3. Fill in squares until you can see a pattern. Try to explain the pattern.Thus far the preparations. It's now your turn with the limited king in square (6, 10).
4. Can you win from that position? If yes, what is your first move?
5. Now put the king in square (1020, 389785). Can you win from that position? If yes, what is your first move?

In the third part the difficulty of tasks increased. Evidence of that can be identified from pupils' solutions. Only two out of five groups solved this problem, one group got correct solution. In the fourth part, pupils can decide between two final tasks. Both of them were complex tasks where pupils should use discovered strategies, understood terminology and systematic investigation to find and explain their solution. Second final task was to determine the winning and losing positions of the queen for the whole field. The allowed moves are in the Picture. The task was more complicated than the previous one. Pupils may find a characteristic description of the losing fields, what was excellent!

- It would also be nice if you can indicate the winning move for the following starting squares: (15, 31); (20, 21); (100, 200).

5 Data collection

The competition was held in Nitra, at the Department of Mathematics FNS CPU with the attendance of 22 pupils from four secondary-grammar schools. They formed six groups. Later on, 18 pupils in five groups from two secondary-grammar schools participated in the competition at the Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava. The teams of three or four pupils worked on an assignment for one whole day. The assignment comprises of one or more open problems with a common context. Pupils can use every accessible method, tool or aid, including the computer and the internet. The result of their work is the solution in the form of coherent mathematical text understandable for every reader, even for someone who does not know already in advance the topic and terms of the

assignment. The participants should also include the description of the solutions, the answers to the questions asked, their logical thinking and reasoning into the report. The solution must be handed in in the electronic version during the competition day, at least seven hours from the beginning of the competition. Pupils worked on the tasks during one day from nine a.m. to five p.m. included short introduction at the beginning, lunch break and writing of the final report.

6 Research question

Within the pilot study, different perspectives were taken. In this article, we focused on the research questions to find out To what extend participated secondary school pupils are able to use inquiry skills for solving open problems from unknown mathematical context.

7 Theoretical framework

Developing of the inquiry processes is the main aim of the Inquiry based learning (IBL). This approach to learning recently spread within Europe through several projects and organisations. Our pilot study is framed within the 7FP project PRIMAS that characterises IBL as an innovative way of learning and teaching [2]. It aims to develop 'inquiring minds and attitudes' of young people which will be required in an uncertain future. Carrying out an experiment in which one tests a scientific hypothesis, developed by drawing on previous knowledge, generally enables understanding of cause-and-effect relationships. This leads to activity in which deliberately defined or selected variables are modified, controlled, monitored, measured, analysed and interpreted. These processes form the inquiry skills that we are about to observe within the results of the competition B-Day.

8 Methods

The main sources of analysis were pupils' final reports, filled questionnaires and semi-structured interviews with participating pupils after the competition. There was quite high percentage of unanswered questionnaires that is why we used them only as a supportive data. We are not presenting a deep description of the pupils' results as well as it was not our aim to monitor all pupils' thoughts and discussions during the competition that may have influenced the results. We presented on few selected examples, the general overview of the spirit of the competition, as well as sufficient evidence that will satisfy our research question.

9 Findings

According to the analysis of the pupils' solutions and information from a questionnaire or interview, we could take the mathematical content as unfamiliar for most of the pupils. The way of working was also new for pupils. On the question whether they did something like investigation they answered: *"It is totally different during mathematics lesson, it is everything so strict."*

Considering the secondary school pupils, all participated groups solved part one. Some of the groups didn't include results from the first and second part into the final report, but there is an evidence in supported materials that they solved the tasks. Solutions from the first part were mostly correct and pupils got good understanding of processes as experimenting, formulating hypothesis, finding conclusion. Another

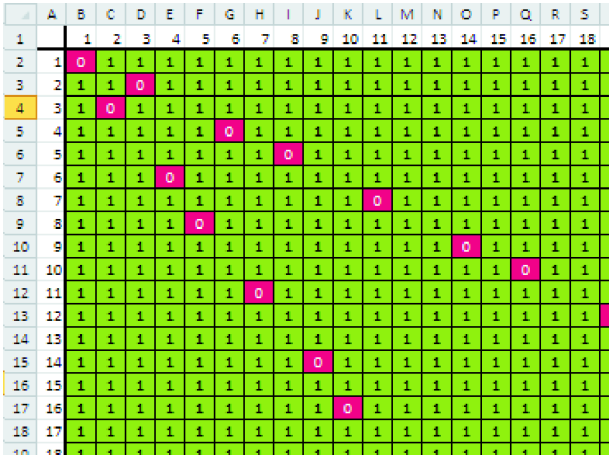

Figure 3: Students' solution

very important aim of the first part was to develop the inquiry processes. The whole competition had several possible directions that the pupils may go through.

In Bratislava none of the five groups chose the first final assignment. In Nitra, there were two out of six groups that solved the first final assignment. None of the groups solved the whole final problem, but most of the groups that solved this task correctly applied the developed backtracking strategy to solve the problem. Most difficult thing was to find a common conjecture that will help to identify general winning strategy.

There have been different kinds of description of the solution that pupils have written in their reports. They for example used excel (See Figure 3). Some of them included detailed description and explanation and some of them included only precisely formulated solution. During the interview, the groups of pupils reflected on the process of how they came up with the solution. We asked them whether they used some strategies that they had known before. They answered: *"We created them today, because we came here without knowing anything about this."*

Eight out of eleven groups have written some solution of the final assignment. They were able to used what they found out during the day, used the terminology and reason mathematically to support they solution. Even though pupils did not work this way based on the day activities, they were able to, in different levels, develop appropriate strategies to solve the problem. We also observe the big advantage of previous knowledge. Most successful pupils were pupils from classes specialized in informatics and mathematics. They differ from other groups in the character of solution. Pupils from informatics class were able to express observed relationship in a recurrent way. The formulas are for one line:

$$X : X(n+1) = x(n) + 2 + a(n)$$

$$Y : Y(n+1) = y(n) + 1 + a(n)$$

They also describe what $a(n)$ stands for. This knowledge was not mentioned in the assignment and also is not common in regular class. That gave us the conjecture

that previous knowledge played also some role in the highest level of the task. There were two groups from both cities that did perform quite poor. They were able to solve some tasks from the first part but didn't manage to go further. The incorrect reasoning was present also in the basics tasks. One group did solve the initial task bud didn't continue with the other parts of the assignment. To clearly understand this, we need more information. Six groups did recognised patterns and gave appropriate reasoning of them. We consider this group as the group of good mathematics pupils that are keen on learning something new not only from textbooks or teacher but through their own investigation. Some problems which pupils were not able to solve by themselves were solved within the peers during a group work. The level of theoretical knowledge wasn't the most important thing. What mattered was the level of inquiry processes and the ability to satisfy their curiosity with investigation. As one pupil expressed during the interview: *"The important thing is that we did not need some..., how to say it.., some deep previous knowledge. Good sense is sufficient and correct judgement. Simply, I have some idea and I try to bring it to conclusion."*

10 Conclusions

The Mathematics B-DAY achieved a significant success among the pupils from both Slovak towns. They considered the problems from the assignment interesting and many of the pupils surprised the organisers by the level of their mathematical skills, by the level of the ability to express their ideas, logical reasoning and also by the results achieved in the final report. Connecting the pupils' solutions with expressed opinions, we see the potential of mathematical competitions like this in Slovakia. There is also a significant evidence of adapting the backtracking strategy in the problem solving process. Even though pupils haven't been used to do similar tasks in the school they were able to work within the inquiry. As we observe all of the pupils were interested in mathematics. That gives us the optimisms that with further preparation and more experience in inquiry processes, the results can be even better. The last group of pupils that didn't manage to overcome the initial problems gave us the information that the competition has its difficulty and not all pupils are able to succeed without the preparation.

Acknowledgements

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