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MODELLING: A FEDERATING THEME IN THE NEW CURRICULUM FOR MATHEMATICS AND SCIENCES IN GENEVA COMPULSORY EDUCATION (AGE 4 TO 15)

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In this paper we show how modelling appears as a federating transversal theme for mathematics and sciences in the new curriculum for all compulsory education in French speaking Switzerland. We then present a training course for in-service teachers we have set up on this theme for all mathematics teachers in Geneva's lower secondary schools, in relation with the European project PRIMAS[1]. Based on a broad definition of modelling, we introduce a typology in three levels of modelling activities. Finally, we show how this was used in this training course in order to classify and analyse a few activities in relation with modelling in the new mathematics textbook for grade 7.
Keywords: Modelling, curriculum, teachers'training.

THE CONTEXT

A new curriculum in French speaking Switzerland

In French speaking Switzerland, a new curriculum (known as *Plan d'Etude Romand*, PER (CIIP 2010)) for all compulsory education (grades -2[2] to 9) has recently been adopted and launched in Geneva, since this year 2011/2012. It divides the school disciplines into five domains, one being *Mathematics and Sciences of the Nature* (MSN). One important novelty concerns the affirmation of modelling as a common federating theme for the whole domain.

The domain is divided into 8 themes: 4 specific to mathematics, 3 to science, wile the middle one, the fifth, is devoted to modelling and is common to mathematics and sciences. It is defined a bit differently in the 3 different cycles with some graduation from cycle 1 to cycle 3, but the core of the presentation is common. In this paper we will focus on the definition given in cycle 3 (grades 7 to 9, age 12-15) (MSN35):

> "MSN35 – Modelling natural, technical or social phenomena or mathematical situations by …
> A) … mobilising graphical representations (codes, schemas, tables, graphs,…)

[1] The project PRIMAS, *Promoting inquiry in mathematics and science across Europe,* has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 244380. This text reflects only the author's views and the European Union is not liable for any use that may be made of the information contained herein.

[2] One important new issue in the PER is the fact that compulsory school now starts at grade -2, age 4. Compulsory education covers Primary School divided into: cycle 1 (grades -2 to 2) and cycle 2 (grades 3 to 6) and lower secondary school: cycle 3 (grades 7 to 9).

B) … assigning to observable magnitudes some parameters
C) … sorting out, organising and interpreting data
D) … communicating one's results and presenting some models
E) … treating of random situations with the use of probability
F) … organising a questioning (dégager une problématique) and/or formulating hypotheses
G) … choosing to apply existing models
H) … mobilising according to the situation, measure or/and some mathematical tools (functions, statistics, algebra,…).” (CIIP2010, inside cover)

Moreover this theme is supposed to contribute to develop transversal capacities such as collaboration, communication, learning strategies, creativity, and reflexive and scientific approach. In the lexical index of the domain MSN, modelling is defined as “the idea of associating to a complex situation a model that makes it comprehensible by reducing it to its essential elements” (Ibid., p. 58).

In parallel with the new PER an inter-cantonal commission has produced a new mathematics textbook[3] for grade 7 (to be followed by one for each grade 8 and 9 in the next years). This textbook is divided in 5 sections, the first four refer to the first 4 themes of the domain MSN while the fifth, called Research and strategies, regroups more challenging activities in terms of problem solving and investigation. Like the previous collection and textbooks fro primary school in mathematics, this textbook is a collection of activities with no formal presentation of theoretical results. These are to be given by the teacher according to her/his own organisation. Nevertheless students have a reminder-book that gives a synthesis of the most important theoretical results for the 3 grades (7-8-9). In this reminder-book, at the end of the last section there is a definition of modelling: “Create a simplified representation of a problem (schema, sketch, table, graph, simulation, etc.), in order to understand and solve it”.

One can see that in this new curriculum the term modelling is taken in a broad meaning, supposed to be symbolic of scientific thinking, federating mathematics and sciences. In this sense, modelling is not only applied to a ‘real concrete’ situation, whose complexity is to be reduced, in order to be treated mathematically. Indeed, it is clearly said that modelling can be intra-mathematical. In this sense, it is closer to a view shared by several researchers in mathematics education, especially within Chevallard's *Anthropological Theory of Didactics*, which claims that “most of the mathematical activity can be identified (…) with a mathematical modelling activity” (Chevallard, Bosch & Gascón, 1997, p. 51 cited in García & Ruiz Hiugeras, 2005, p. 1647, see also Artaud, 2007).

A training-course within the European project PRIMAS[1]

The new PER and the new textbook for grade 7 led the educational authorities in Geneva to organise a one-day (compulsory) training course for all mathematics teachers of grade 7-8-9 (around 350 teachers, half of them also teaching some science). As part of the European

[3] Since 1972 for primary school and 1995 for lower secondary schools in all French speaking Switzerland there are some official textbooks in mathematics. They are written by a specific commission mandated by the inter-cantonal authorities and are supposed to be the only resource for teachers and students.

project PRIMAS our team of 15 teachers, teachers' trainers, researchers in mathematics and science education was contacted to organise this training. Due to a long tradition in the curriculum of problem-solving, most mathematics teachers in Geneva secondary schools have the feeling that they already know what problem solving and modelling mean. Yet, they are in several cases reluctant to teach them because it is time consuming and they have a huge pressure to finish the programme, in terms of content, even if it is reduced to technical tasks. This is also less popular with students, as was already pointed out by Blum and Niss:

> "Problem solving, modelling and applications to other disciplines make the mathematics lessons unquestionably more demanding and less predictable for learners than traditional mathematics lessons. Mathematical routine tasks such as calculations are more popular with many students because they are much easier to grasp and can often be solved merely by following certain recipes, which makes it easier for students to obtain good marks in tests and examinations." (Blum & Niss 1991, p. 54)

One day training is short, too short to be able to change things radically. Moreover, experience with in-service teachers' training has shown that valuable training must be practical, close to teachers' concerns and aware of their potential of evolution without trying to be too radical. In our case, we tried to take these concerns into account by setting up a training-course to show that investigation and modelling can be taught in a somehow modest version without consuming too much time and necessitating other resources than what is offered in the official textbook. On the other hand, we focused on the role of the teacher while piloting an activity in class. Our aim was to give some useful tools in order to choose during an activity when it is necessary to help students organising their questioning or, on the contrary, when it is essential to let students find their own way through the solution in order to keep up the potential of investigation and/or modelling of the situation.

A TYPOLOGY FOR MODELLING ACTIVITIES

Our first concern was to have a definition of modelling that could be both broad enough to respect the PER and, at the same time, operational in order to analyse the different activities that could be seen as modelling. Indeed, modelling being a transversal theme, no specific activities are pointed out as modelling activities in the textbook, it is the teachers' responsibility to choose how to give the modelling orientation.

A definition of modelling

In the literature on can find several definitions of what is modelling, we decided to adopt a very broad one, which is not very far from the one proposed by Niss, Blum and Galbraith (2007, p. 4) in the introduction of the 14th ICMI Study on Modelling and applications in mathematics education, yet we enlarged it by erasing the condition that one domain is non mathematical and the other one is mathematical. In this sense our definition can involve two mathematical domains or, on the contrary, two non-mathematical domains (like this is used in biology teaching quite often).

Modelling means setting up/discussing/tackling a correspondence (mapping) between two (or more) systems4 including objects, relations between these objects and questions.

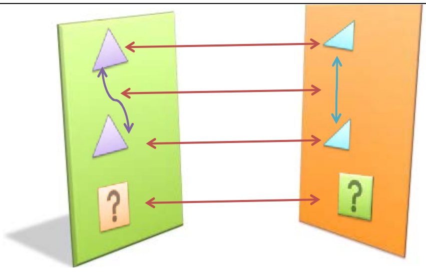


Figure1. Schema for modelling

In the "traditional" mathematical modelling, the first system is extra-mathematical, while the second one is mathematical. Moreover, the most important thing in the mapping is to reduce the complexity of the initial system in order to be able to solve the questions in the model and then be able to translate the solutions in terms of the first system. Like Niss, Blum and Galbraith point out this process is dialectical and may need more than one back and forth passage to adjust the mathematical model.

We preferred the term of system to domain as used by Niss, Blum and Grabaith. One reason is that the term 'domain' may induce the fact that one is extra-mathematical and the other one mathematical, while system can be seen as more neutral. Another reason is to point out the fact that we deal with objects and relations between them. In this sense, we haven't included in the systems phenomena and assumptions like the authors of the ICMI study. Indeed in our approach, these can be interpreted in terms of objects and relations and questions about them. In other words, we have chosen a definition broad enough to embrace all the variety of situations that can be found in the curriculum of mathematics and sciences in all compulsory education in Geneva.

A typology in three levels

The most challenging way of engaging students in modelling is to give them a problematic question in a system and to leave them to build another system (the model) in which this question can be solved. In this case, students have the responsibility of reducing the complexity of the first system to some significant elements, choose the second system and build the mapping, solve the problem in the second system and interpret the solution in terms of the first system. All this is described in many papers about modelling, in particular in the introduction of the ICMI study quoted above. However, in some teaching situations, the two systems may be given and what is expected from the students is either to interpret part of one system in terms of the other or to discuss the validity of the correspondence in relation to a certain type of question (in particular more than two systems can be given and the correspondences should be compared in terms of consistency). These types of activities, even if less challenging, may still offer a good opportunity to engage students into a rich reflection about modelling, in particular it is often the case in science teaching in Primary school. Finally in some situations, the two systems are given but most of the students' task is to solve the question in the second system, without real reflection about the relation with the first one.

[4] We used term system instead of domain in order to avoid the reference to domain of reality and mathematical domain and also in order to point out the question of relations between objects.

Most of the time, this type of activity does not present much challenge in terms of modelling, the initial system is only evoked, the mapping with the other system is transparent and the modelling acts, at most, as an extra motivation for students.

Based on these remarks, we proposed a typology of modelling activities in 3 levels:

- Level 1 – The two systems are given but the task given to students involves only the second system
- Level 2 – The two systems are given and the task involves the two systems
- Level 3 – Only one system is given and students have to choose the other(s)

In our training course we presented this typology to the trainees and we ask them to use it to classify a selections of 12 activities taken from the new textbook for grade 7. Then we made a more specific analysis on three of them focusing on the teacher's work to realise a maximum of the potential of modelling and investigation for the students. We will now briefly show how we organised our analysis

SOME EXAMPLES TAKEN FROM THE NEW MATHEMATICS TEXTBOOK FOR GRADE 7

We are now presenting how we can apply this typology on some examples.

As a first example of level 1, here is an activity about the search of the centre of the circumscribed circle of a triangle:

Find a point of observation located at equal distances from the road, the railway and the river.

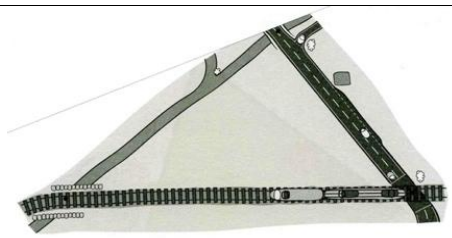


Figure 2. ES28 – Point of observation[5] (CIIP 2011b, p. 118)

Here it is clear that the context of the real situation does not change much compared to the basic task in which only a triangle would be given. At the maximum one can say that the real situation gives an extra motivation for the mathematical task.

Let us come now to examples of level 2.

Among the following plane patterns of solids, which correspond to right prisms? Rectangle parallelepipeds?

[5] This is the way activities are designated in the textbook: the two letters refers to one of the 5 themes (NO numbers and operation, FA functions and Algebra, ES space, GM magnitudes and measures, RS research and strategies), the number is the order in each theme and the title refers to the task.

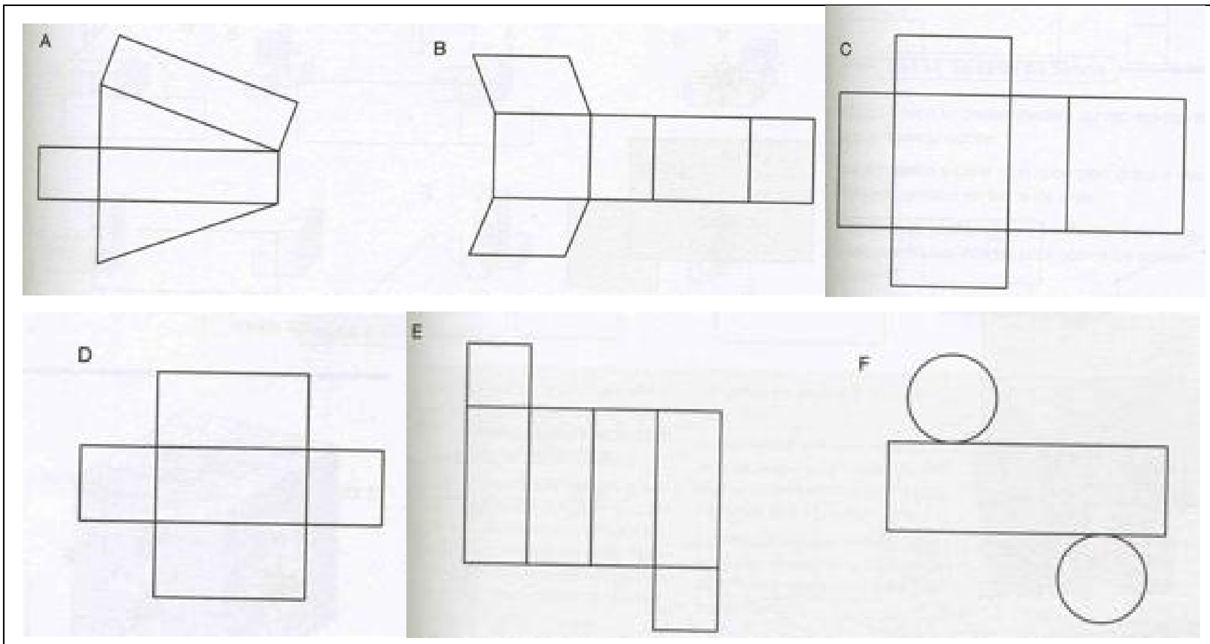


Figure 3. ES107 – From one pattern to another (CIIP 2011a, p. 121)

In this activity, the two systems at stake are the plane patterns (2D) and the solids (3D). This is an intra-mathematical modelling. One way to answer the questions is to actually build the patterns, form the solids and check on the solids which type they are. Another way consists in imagining this, without doing it really. Furthermore, one can try to characterise the type of patterns for a right prism, i.e. two identical or symmetrical faces (the top and bottom of the prism) and as many rectangles as sides of the top of the prism with corresponding dimension, the other dimension being identical for all the rectangles (the height of the prism) (one still need to check that the pattern can actually form a solid). In order to set up these conditions, one has to develop a dialectical exploration between the system of plane patterns and the system of the solids. This is a way to exploit the potential of modelling in this activity and to make it a bigger challenge for students.

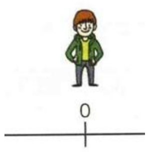
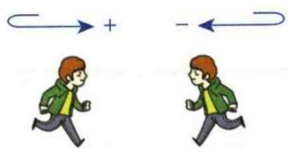
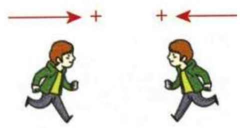
With the help of the little moving man[6], give the result of the following operations: a) 0 + (+3) e) (+8) + (+4) b) 0 + (-3) f) (-7) + (-2) c) (-4) + (+5) g) (-4) + (+2) d) (+3) + (-5) h) (+7) + (-2)	The little man is at rest 	He looks in the direction of positive negative 	He moves 

Figure 4. NO119 – moving forward (CIIP 2011a, p. 38)

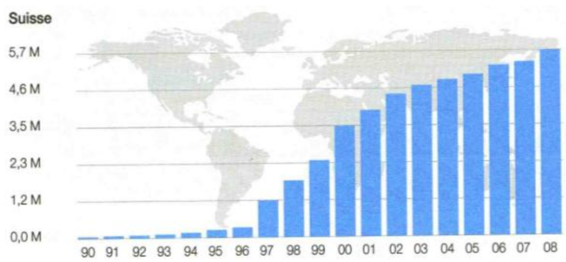
Here, the modelling is not intra-mathematical, even if the system with the little man moving along the line is not quite real, more a didactical artefact. However, this system refers to

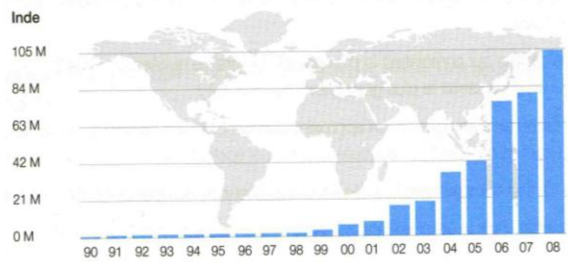
[6] An example is given with a little man moving along a directed line with graduations.

movement along a line, half turns and equal steps in one or the opposite direction, which refer to a certain reality of space. This type of model is quite effective to understand the relation between negative and positive numbers (level 2). Unlike models with thermometers and elevators, often used in class, it does not present the negative numbers as something attained by getting under zero (zero being seen as a repelling point) but rather like a symmetrical extension of the positive numbers (zero as an articulating point). It is interesting to note that a presentation of negative numbers based on these ideas appears at the very beginning of Argand's treatise on the geometrical representation of imaginary quantities (our complex numbers) (Argand, 1806). In this paper it is the key idea for Argand to understand that $\sqrt{(-1)}$ can be represented by one step in an orthogonal direction from the line of real numbers (either at the top (+i) or the bottom (-i)). His genius is to interpret $\sqrt{(-1)} = \sqrt{(+1)(-1)}$ as the geometrical mean between (+1) (one step on the right) and (-1) (one step on the left).

Let us come now to level 3

This diagram shows the evolution of the number of Internet users in Switzerland and India.





a) Evaluate by what factor the number of Internet users has been multiplied, in Switzerland and in India, between 2003 and 2008. How can you explain this difference?

b) Considering the fact that the population in Switzerland was roughly 7.7 millions inhabitants in 2008, would you say that many or few inhabitants of our country have access to Internet in our country? Why?

c) Considering the fact that the population in India was roughly 1.15 billions inhabitants in 2008, would you say that many or few inhabitants of this Asiatic country have access to Internet? Why?

d) With the help of the two diagrams, imagine the number of Internet users there will be in Switzerland and in India in 2020. Give some justifications.

Figure 5. FA54 – Internet (CIIP 2011a, p. 84)

The first three questions are very similar to what we have showed in the preceding example and illustrate activities of level 2. The last question though is quite different. Indeed it is a prospective question that cannot be tackled only by looking at the graph. It necessitates discussing another type of modelling about the evolution of Internet users in the future. Therefore it implies choosing a new system and is a task of level 3. It is very usual for students in such cases to apply a linear model. Here, question a) led to find out that the number of Internet users between 2003 and 2008 has increased from 4.7M to 5.7M in Switzerland, so an average increase of 0.2M users per year, and from 20M to 104M in India so an average increase of 16.8M users per year. Therefore a linear extrapolation would lead one to think that the number of Internet users in 2020 would be 8.7M in Switzerland, which exceeds the total of inhabitants in Switzerland in 2008 (what about in 2020?) and 305.6M in India. On another hand, the way question a) is formulated induces a model of geometrical growth. The number of Internet users between 2003 and 2008 has been multiplied by roughly 1.2 in Switzerland and 5.2 in India. In a geometrical model of growth these data would lead to say that the number of Internet users in 2020 would be roughly $5.7(1.2)^{12/5} \approx 8.8$M in Switzerland and $104(5.2)^{12/5} \approx 5438$M in India, which exceeds by far the population of this country in 2008 (and very likely in 2020). Another way to tackle the question would be to draw a curve that follows at best the pattern of evolution. Here the question of the model of growth is very open and more challenging. This is clearly what we have identified as level 3, when the second system has to be discovered by the students with a discussion on its validity.

Ana measured 1.37m when she was 11 and 1.45m the following year. What will be her height when she is 16?

Figure 6. FA33 – Growing (CIIP 2011a, p. 74)

This is a very similar task than the last question of the preceding example, even if it is a bit less challenging. A linear model can be easily put to the question if one interpolates in a further future, for instance when Ana is 30.

Peter draw a square with size 6cm and ask his daughter Nathalie to divide it into 9 square pieces with a side measured by a whole number of centimetres.

Nathalie finds quickly one possible division but asks herself whether there are any other ones. Two division made by the same squares, organised differently are considered identical. How many divisions are there?

Figure 7. RS1 – Divided square (CIIP 2011a, p. 169)

The most "natural" way to tackle this problem is to try to organise a division by placing squares into the big square. One can start by putting a 5x5 square, then 4x4, etc. It is not too complicated this way to discover three solutions:

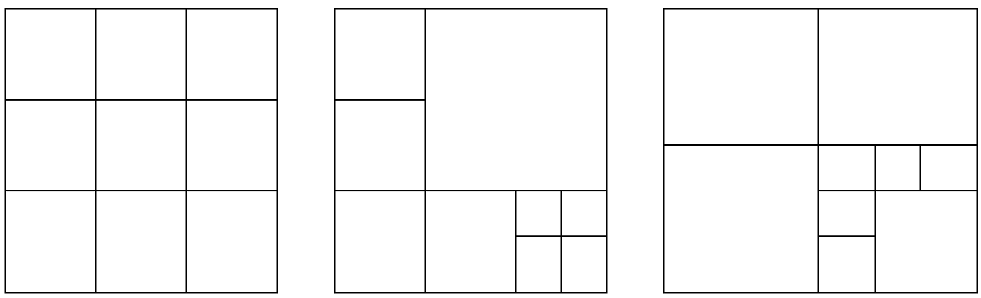


Figure 8. – The 3 solutions of problem RS1

The main difficulty is to be sure that they are the only three. That can be justified as we mentioned above by a systematic exploration, but it is difficult to write the whole justification. Another way to achieve this justification is to explore the problem numerically, which means translating the problem of squares in a new system involving numbers, in other words modelling the geometrical situation into arithmetical terms. The task becomes to list all the possible decompositions of 36 in a sum of exactly 9 squares of whole numbers (1, 4, 9, 16 and 25). This can be done and justify quite easily and apart from the 3 solutions corresponding to the divisions above, one finds another one: 36 = 25 + 4 + 1 + 1 + 1 + 1 + 1 + 1 + 1.

But this decomposition, unlike the other three does not correspond to an actual division of the square, indeed if one puts a 5x5 square, there will be no space for a 2x2 square!

This very brief mathematical analysis shows the virtue of the modelling by arithmetic in order to give a rigorous proof of the fact there are only three ways of dividing the square in 9 squares. Our experience with students of grade7 is that they do not think of an arithmetical model and moreover, do not feel the necessity to prove rigorously the fact that apart from the 3 solutions everybody agrees on, there is no other, since precisely nobody in the class has found another one!

One can try to encourage them to prove it, but they will just try in order to please their teacher not because they feel a necessity for it. In the same way, bringing the arithmetical modelling on the scene is also very likely to be taken as a teacher's magic trick. In other words, there is a dead-end here due to a resistance of the didactical contract. Following Brousseau's work on the contract and the milieu, Perrin-Glorian and Hersant (2003) have shown the complementarity of these concepts. Here, we have a good example of this complementarity. Indeed since the situation cannot evolve without forcing a new contract, one can modify the milieu in order to make the change of contract effective. In our case, the idea is to add a new information in the milieu: "Nathalie has discovered the three solutions above. Peter agrees, but says that a friend of his has discovered a solution with two squares with side 3, four with side 2 and three with side 1."

This solution with 9 squares does not work. Moreover, it is easier to invalidate it by adding the different sizes of the squares that makes 37cm^2 and not 36 cm^2, than by trying to organise the actual division of the square. In other words, bringing this new information in the milieu creates some doubt that forces the students to check that this is not another solution and it is also proper to show the benefit of an arithmetical approach (even if not all the students think about it, it is enough that one student produces it, to show the benefit), which are changes in the didactical contract.

At this stage, one can bring another information in the milieu: "Ok this solution does not work, but somebody has found a solution with one square with side 5, one with side 2 and seven with side 1". Like we said above, this time the arithmetical argument cannot invalidate the solution, but it is easy to see that it cannot be realised.

CONCLUSION

The aim of the training-course is to help teachers taking at best into account the requirement of the curriculum concerning modelling and investigation. Our categorisation in 3 levels is a tool in order to analyse the potential of modelling in an activity. In the training-course, we have also worked on the actual realisation of such an activity in class and discuss some ingredients of teacher's work. We have used videos of real class activities, especially concerning the divided square. Indeed however the activity is a good modelling activity the question of how it is implemented in the class and how much the students are involved in a real investigation is crucial. In everyday teaching, teachers need some tools to be able to regulate their way of piloting such an activity. Example like what we presented very briefly above about the dialectical use of milieu and didactical contract in the activity of the divided square is one of our leading orientation in our actions within PRIMAS in order to promote investigation in mathematics and sciences especially through modelling.

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