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**Which is harder? - Classification of Happy Cube puzzles**

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# Which is harder? - Classification of Happy Cube puzzles

## 1. What is a Happy Cube?

Happy Cubes are a set of mechanical puzzles created in 1986 by the Belgian toy inventor Dirk Laureyssens and they are also known as "Snafooz" in USA, "Wirrel Warrel" in Netherlands or "Cococrash" in Spain ([1]). A Happy Cube consists of 6 foam pieces as in figure 1 and 3.



FIGURE 1. A Happy Cube - as in the initial packing



FIGURE 2. A Happy Cube - 3D mission completed

There are three main tasks: the 2D mission is to fit back the pieces into the frame (as in figure 1), the 3D mission is

to build a cube with the given pieces (as in figure 2) and the  $\infty$ D mission is to build 3D objects using the pieces of several puzzles. The 2D mission is easy for all the cubes and the solution is given by the initial packing while the  $\infty$ D mission depends on too many parameters (the shape of the object we want to build and the sets we use). For this reason we analyze the 3D missions.

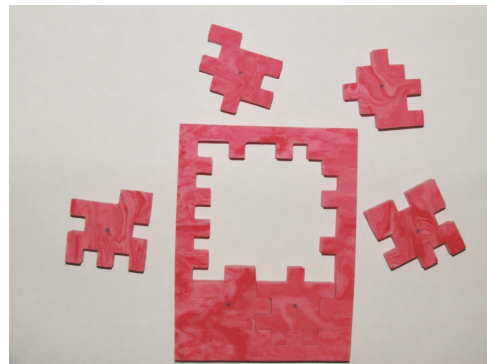


FIGURE 3. The pieces of a Happy Cube

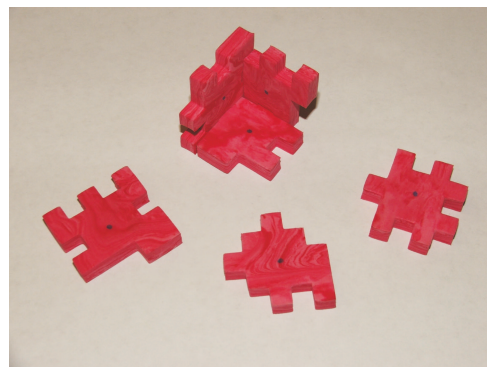


FIGURE 4. 3 pieces in a 3D configuration

Usually there are a lot of possibilities to join the first 2 or 3 (or even 4 or 5) pieces (see figure 4) or figure 5)

but in many cases the last piece does not fit the existing configuration.

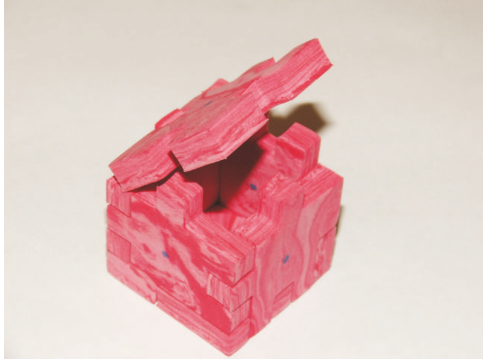


FIGURE 5. The moment of truth: does it fit?

The Happy Cube Family is divided into 4 different sets: Little Genius, Happy Cube, Profi Cube, Marble Cube. According to the manufacturer's homepage ([2]) these sets have different difficulties and the difficulty of each set is marked with the corresponding number of stars as follows:

- Little Genius ★
- Happy Cube ★★
- Profi Cube ★★★
- Marble Cube ★★★★★

Each of these sets contains 6 different models, labeled also with stars accordingly to their difficulty. The Profi Cube and the Marble Cube set consists of the following cubes:

	Profi Cube	Marble Cube
★	Confusius	Martin L. King
★★	Da Vinci	Omar Khayam
★★★	Marco Polo	Marie Currie
★★★★	Rubens	Buckminster Fuller
★★★★★	Watt	Mahatma Gandhi
★★★★★★	Newton	Albert Einstein

The Profi Cube set is recommended for the age-group 7–125 and is considered "Difficult" while the Marble Cube set is recommended for the age-group 9–125 and is considered "The most difficult."

We have been using these puzzles (the Profi Cube set and the Marble Cube set) for several years not only as an educational toy but also as teaching material in our mathematical modelling activities. These activities were designed for 14–18 years old students and were focused mainly on developing the concept of graph (and hypergraph), the "rediscovering" of the backtracking algorithm and the optimization of backtracking algorithms using cutting conditions. During these activities we (and our students) observed that something is wrong with the difficulty ranking given by the manufacturer. Our impression was that the Marble set is easier than the Profi set and even inside of a set the stars do not contain any relevant information on the difficulty of the model. So the following natural problems arose:

- (1) How to construct a proper ranking of the cubes by measuring the required time for solving the puzzle (separately for each cube)?
- (2) How to define a theoretical complexity for these cubes such as the classification determined by this complexity fits the previous ranking?

## 2. A little mathematics and algorithm analysis

We represented each face of the cube as an  $5 \times 5$  matrix with elements 1 and 0. The element in the  $i^{th}$  row and  $j^{th}$  column is 1 if and only if the corresponding square of the face is full (see figure 6).

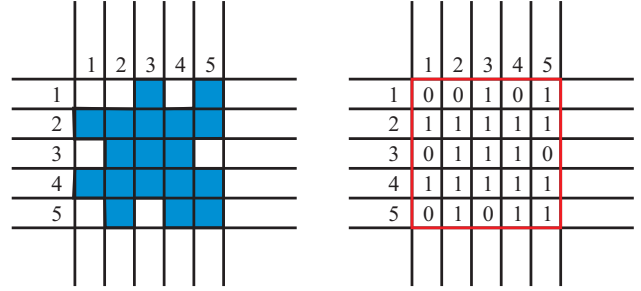


FIGURE 6. Representation of a face

To handle the rotations of a face we used 8 such matrices for each face (4 different positions obtained by rotations and the symmetrical of these positions) and we denoted these matrices by  $M_1, M_2, \dots, M_{48}$ . Depending on the symmetry of a face among these matrices we can have several identical matrices but this is not a problem. We define a relation  $\sim$  on the set of these matrices in the following way:  $A \sim B$  if and only if  $A$  and  $B$  correspond to different pieces and the corresponding pieces can be joined such that the common edge of the pieces is described by the first line of the matrices  $A$  and  $B$ . Technically the matrices  $A$  and  $B$  corresponding to different pieces belong to the same equivalence class if and only if

$$a_{1i} + b_{1i} = 1, i \in \{2, 3, 4\};$$

$$a_{11} + b_{11} \leq 1 \text{ and if } a_{11} + b_{11} = 0, \text{ then } \min\{a_{21}, b_{21}\} = 0;$$

$$a_{15} + b_{15} \leq 1 \text{ and if } a_{15} + b_{15} = 0, \text{ then } \min\{a_{25}, b_{25}\} = 0.$$

In this way the solution of a cube can be described by a set of 12 corresponding pairs. For this reason in the first step we generated a  $48 \times 48$  incidence matrix denoted by  $F$ . The element  $f_{ij}$  of this matrix is 1 if the corresponding pieces fit each other ( $M_i \sim M_j$ ) and 0 in the other case. Clearly the sum of the  $j^{th}$  column in  $F$  gives the number of pieces that fit the corresponding piece at a fixed edge. In a solution any piece has its place so, we can start the assembling of the cube from any piece. Hence we can choose the first piece (and position) coded by the matrix  $M_j$  such

that the number  $\sum_{i=1}^{48} f_{ij}$  is minimal. This ensures that even if we do not choose the right pieces at the beginning we have to try a minimal number of possibilities. In a very lucky case if this minimal number is 1, we have a certain

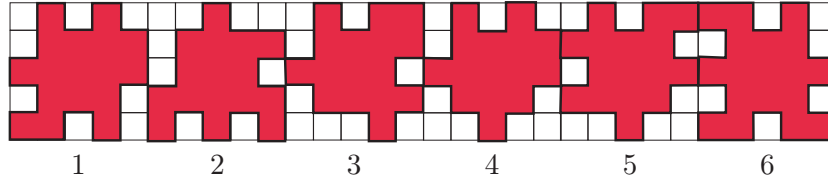


FIGURE 7. The pieces of the Watt Cube

starting point. This situation occurs in the case of the Watt Cube (see figure 8 and the other pieces in figure 7).

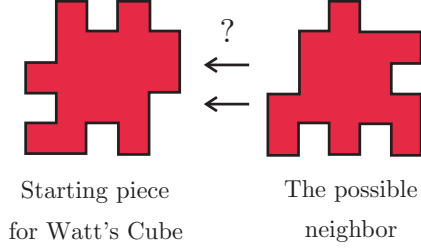


FIGURE 8. A certain start for the Watt Cube  
We denote by  $c_1$  the minimal column sum, so

$$(1) \quad c_1 = \min_{1 \leq j \leq 48} \sum_{i=1}^{48} f_{ij}$$

is the first complexity indicator. In the following table we have the first complexity indicator for the analyzed cubes:

Profi	$cs_1$	Marble	$cs_1$
Confusius	8	King	2
Da Vinci	8	Khayyam	8
Marco Polo	7	Curie	3
Rubens	6	Fuller	2
Watt	1	Gandhi	2
Newton	2	Einstein	3

Some of the cubes have symmetrical pieces and this must be taken into account. This can be done if in the initial incidence matrix  $F$  we introduce fractions. If a piece  $L$  (which is coded by the matrices  $M_{i_1}, M_{i_2}, \dots, M_{i_8}$ ) has some kind of symmetry, than for each  $M_{i_k}$  we count the number of matrices in the set  $\{M_{i_1}, M_{i_2}, \dots, M_{i_8}\}$  identical to  $M_{i_k}$ . If this number is  $s_k$ , than we use  $\frac{1}{s_k}$  in the incidence matrix instead of 1 in the rows (and columns) of  $M_{i_k}$ . Let's denote by  $FS$  the so obtained incidence matrix and by  $cs_1$  the corresponding complexity indicator

$$(2) \quad cs_1 = \min_{1 \leq j \leq 48} \sum_{i=1}^{48} f s_{ij}$$

For the sake of accuracy we call  $cs_1$  the first corrected complexity indicator.

In the following table we have the first corrected complexity indicator for the analyzed cubes:

Profi	$cs_1$	Marble	$cs_1$
Confusius	4	King	2
Da Vinci	4	Khayyam	3
Marco Polo	4	Curie	3
Rubens	5	Fuller	2
Watt	1	Gandhi	2
Newton	2	Einstein	3

In what follows we define recursively 3 more complexity indicators by constructing a graph which illustrates the possibilities we have when assembling a cube. For a better understanding we construct this graph for the Watt Cube. If we start by joining piece 1 and 2, then at one side we have 2 possibilities while on the other side there are 3 different possibilities to continue with a third piece (figure 9). We choose the minimum number of further possibilities and for each such possibility we analyze the configuration we obtain by joining the third piece to the first two. In each step we have a few positions to add the next piece and to each position we can associate the number of pieces which can join the existing construction at that position. If we choose in each step the position with a minimal number of further possibilities, we obtain the graph from figure 10 for the Watt Cube.

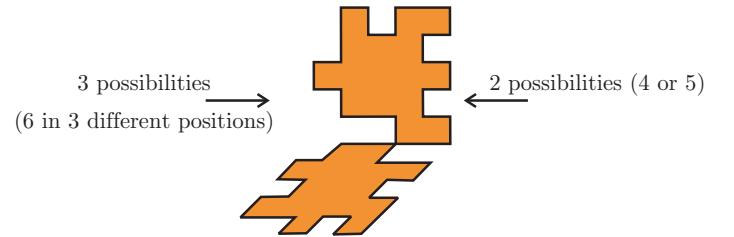


FIGURE 9. The second step in assembling the Watt Cube

Figure 10 shows that if we proceed by a backtracking algorithm we can assemble the cube in at most 3 attempts (the order of the pieces can be 1 – 2 – 5, 1 – 2 – 4 – 3 or 1 – 2 – 4 – 5 – 3 – 6).







FIGURE 11. Puzzle solving activities

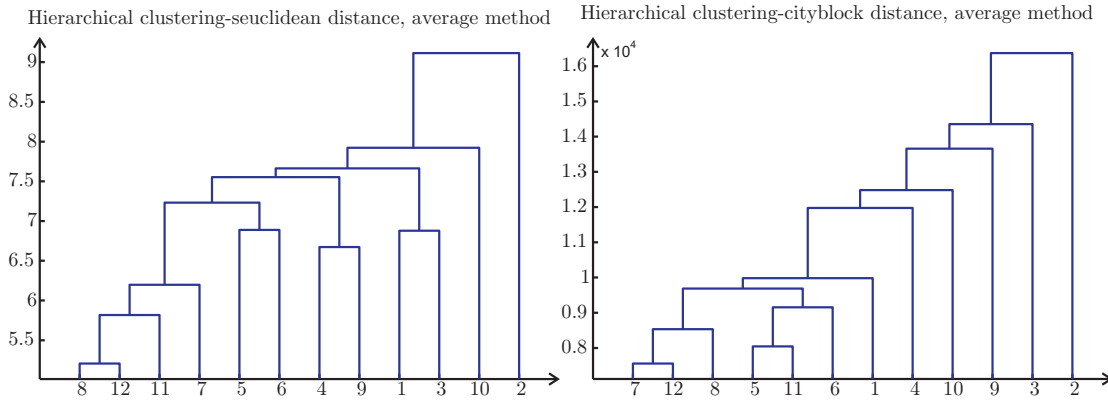


FIGURE 12. Dendrograms (different distances and methods)

Confusius or for the Rubens cube shows that most of the solvers do not observe the certain start configuration and for this reason they had to try more possibilities. Analyzing the measured times for the Watt cube we can observe that they fit an asymmetric bimodal distribution hence the average is not significant for this cube. These observations and the above table shows that the most natural division into two sets is the following:

- I. set (the easy set): 8,12,7,11,5,6
- II. set (the hard set): 1,4,10,9,3,2

We denoted by  $(a_i)_{1 \leq i \leq 12}$ ,  $(b_i)_{1 \leq i \leq 12}$ ,  $(c_i)_{1 \leq i \leq 12}$  and  $(d_i)_{1 \leq i \leq 12}$  the above rankings and we performed a factor analysis between the complexity indicators we defined in section 2 and these rankings.

#### 4. Correlation between theory and practice

By performing a rank correlation analysis (using Kendal and Spearman type coefficients) and a principal components analysis we obtained that:

- the average ranking is best correlated with the ranking obtained from the total number of vertices in the corresponding Greedy graph;
- if we exclude the outliers, the average ranking is best correlated with the ranking obtained from the sum of the first two complexity numbers;

- the median ranking is best correlated with the ranking obtained by using the first corrected complexity number (and the sum of the first two complexity numbers);
- the order obtained from clustering is best correlated with the ranking obtained by using the average number of steps.

In all the cases about 50% of the variance can be explained by a single theoretical ranking and all the theoretical rankings cover 75% – 85% from the variance of an empirical order.

#### 5. Concluding remarks

- The rankings obtained from the puzzle solving activities are in accordance with the ranking obtained by evaluating the corresponding graphs constructed by using a Greedy algorithm. Moreover, most of our students confirmed that they used some kind of trial-error, backtracking and Greedy steps in solving these puzzles.
- The categorization used by the manufacturer is completely wrong. The marble set is not the hardest one and the order of cubes used by the manufacturer within the Profi and Marble sets is not correlated with the complexity of the cubes.

- We suggest the following order of the cubes (the cubes of the second set are harder than the cubes of the first set):

I. set: Omar Khayyam, Mahatma Gandhi, Albert Einstein, Watt, Newton, Martin Luther King

II. set: Marie Curie, Buckminster Fuller, Confucius, Marco Polo, Da Vinci, Rubens

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## 7. Appendix 1

In figures 13 and 14 we illustrated all the possible (non equivalent) solutions of the Profi and Marble cubes. The Omar Khayyam cube is the only cube with several non equivalent solutions.

## 8. Appendix 2

Figures 15 and 16 illustrate the constructed graph for each cube.

## References

- [1] [http://en.wikipedia.org/wiki/Happy\\_Cube](http://en.wikipedia.org/wiki/Happy_Cube)
- [2] The official site of Happy Cube: <http://www.happycube.com/>
- [3] Brian S. Everitt, Sabine Landau, Morven Leese, *Cluster Analysis*, Wiley, 4th edition (January 20, 2009)
- [4] I.T. Jolliffe: *Principal Component Analysis*, Springer, 2nd edition (October 1, 2002)

<sup>1</sup>Promoting inquiry in Mathematics and science education across Europe, Grant Agreement No. 244380

<sup>2</sup>Developing Quality in Mathematics Education, for more details see <http://www.dqme2.eu/>

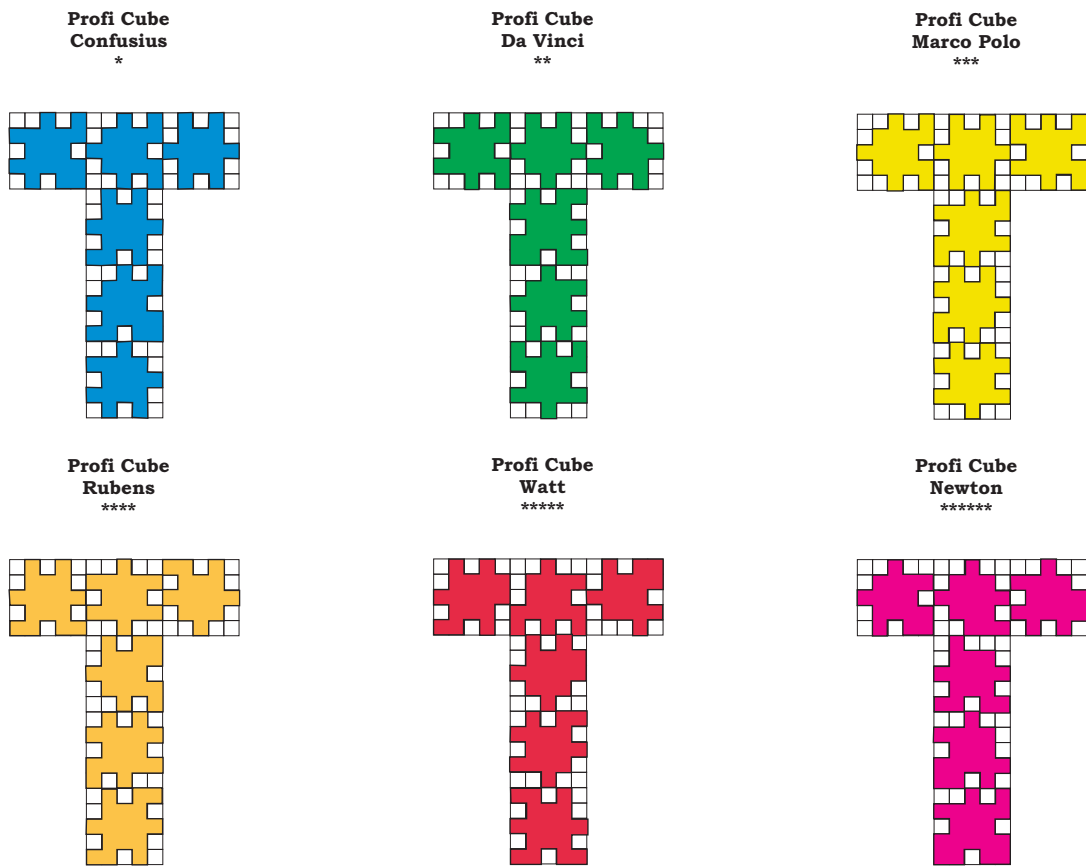


FIGURE 13. Solutions for the Profi Cube



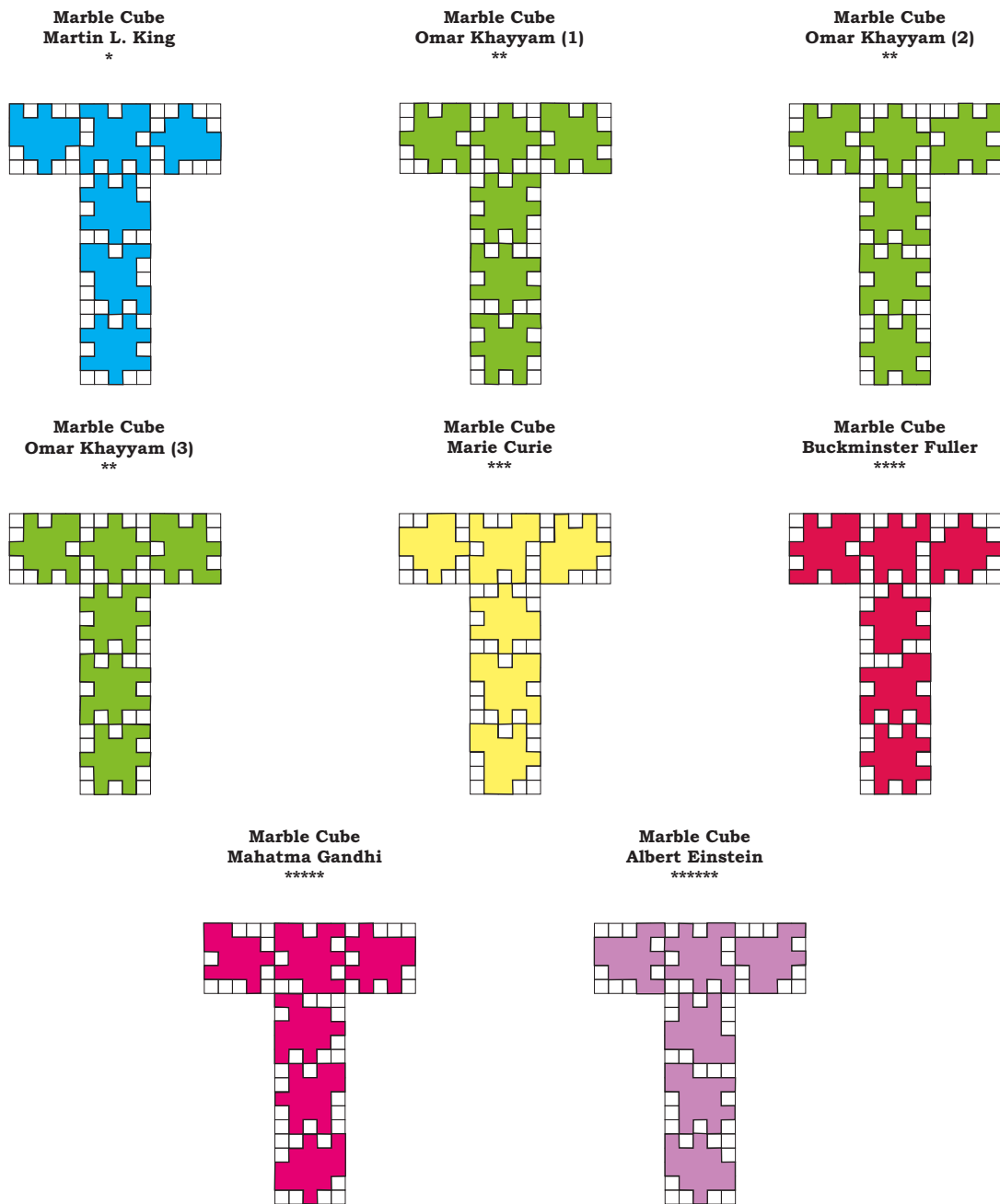


FIGURE 14. Solutions for the Marble Cube



