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# Determination of the Size and Depth of Craters on the Moon

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Experimental work in the research of astronomical phenomena is often difficult or even impossible because of long-lasting processes or too distant objects and correspondingly too expensive equipment. In this paper, we present an example of observation of the Moon, which is our nearest astronomic object and therefore does not require professional astronomic equipment for observation. We focus on the observation of craters on the Moon, determining their lateral size and depth on the basis of photographs and simple calculations. The fieldwork with students of junior grade school education was performed within the framework of the optional subject Astronomy. An analysis of the results of the students' experimental work, as well as of curricula on various levels of education, led us to conclusion that this kind of experimental work is suitable for incorporation in secondary school physics education. With some mathematical simplifications, however, the treatment of the topic can also be appropriate in primary school. Such experimental work enables students to gain specific natural science and mathematical competences that are also required for the study of other natural phenomena.

**Keywords:** astronomic observations, craters, the Moon, natural science competences

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# Določitev velikosti in globine kraterjev na Luni

Vladimir Grubelnik, Marko Marhl in Robert Repnik

Pri preučevanju astronomskih pojavov je eksperimentalno delo zaradi časovno predolgotrajnih procesov oziroma preveč oddaljenih objektov in s tem predrage astronomske opreme velikokrat oteženo oziroma nemogoče. V prispevku predstavljamo primer opazovanja Lune kot nam najbližjega astronomskega objekta, pri čemer ne potrebujemo profesionalne astronomske opreme. Osredinimo se na opazovanje kraterjev na Luni, pri čemer na podlagi posnetih fotografij in preprostih računskih operacij določimo velikost in globino kraterjev. Z namenom preučevanja, kako so posamezne faze eksperimentalnega dela primerne na področju izobraževanja, smo spremljali delo študentov razrednega pouka pri izbirnem predmetu astronomija. Na podlagi analize rezultatov eksperimentalnega dela študentov in analize učnih načrtov na različnih stopnjah izobraževanja smo ugotovili, da je omenjeno eksperimentalno delo primerno vključiti v pouk fizike na področju srednješolskega izobraževanja. Z določenimi matematičnimi poenostavitvami pa je obravnava problema primerna tudi v osnovni šoli. Poudariti velja še, da si učenci prek obravnavanega eksperimentalnega dela pridobivajo tudi nekatere naravoslovno-matematične kompetence, ki jih potrebujejo pri preučevanju drugih naravnih pojavov.

**Ključne besede:** astronomska opazovanja, Luna, kraterji, naravoslovne kompetence

## Introduction

Experimental work in connection with the study of astronomic objects and phenomena often requires expensive astronomic equipment. Besides insufficient knowledge about complex working methods, this is the main reason for the lack of experimental work of this type in the educational area. In this paper, we present an example of experimental work using a telescope, where relatively simple astronomic equipment is sufficient (Astroshop.eu, 2017; Telescope: Meade ACF-SC 203/2000 8" LX200; Astro camera: The Imaging Source DFK), making it financially accessible for primary and secondary schools. The focus is on measurements of the diameter and depth of craters on the Moon's surface.

Craters on the Moon were first observed by Galileo Galilei, who made his first telescope in 1609 and used it to observe the Moon. He discovered that, contrary to general opinion at that time, the Moon was not a perfect sphere, but had both mountains and cup-like depressions, the latter of which he named craters. The word crater was adopted by Galileo after the Greek word for vessel: Κρατήρ – a Greek vessel used to mix wine and water.

Nowadays, there are numerous photographs, catalogues (Wang, Cheng, & Zhou, 2015) and generally available free applications, such as the Virtual Moon Atlas (Legrand & Chevally, 2012), that enable access to a great deal of data about the individual craters on the Moon. In the present paper, we focus on the determination of the diameter and depth of the Moon's craters; however, our aim is not only to obtain these data, but also to study how different phases of the corresponding experimental work suit treatment at different educational levels.

A crater's diameter and depth, or their ratio, are relevant parameters that indicate (within certain limitations) the conditions of the impact of a spatial projectile with the Moon's surface, conditions that influenced the formation of the crater (Scott, 2013). Several studies have been completed (Scott & Toalster, 2002; Scott, 2002, 2013; Scott, Shen, Mulley, & Pan, 2013) that illuminate the question of how to incorporate the problem of crater formation and the corresponding diameter-to-depth ratio in the curriculum for different educational levels. The focus of these studies is the connection between different subjects, in the sense of the scientific approach to solving problems related to the real world, whereby the students can compare their own findings with statistical data from professional scientific research (Scott & Toalster, 2002; Scott, 2002). Suggestions are offered regarding how we can, together with gifted students at different educational levels, determine the individual characteristics of craters with the help of adequately adapted theoretical derivations (Scott, 2013; Scott, Shen, Mulley, & Pan, 2013), as well as with experimental work in the classroom (Scott, Shen, Mulley, & Pan, 2013). This exposes the use of scientific data that can serve as a starting point for theoretical derivations or as a verification of various conclusions. In the case of the study of craters on the Moon, data published by Pike (1976) in a catalogue of crater dimensions, compiled mostly from panoramic photographs taken on the final three Apollo flights, is mentioned several times.

The aforementioned studies focus on work in the classroom. In this area, however, there is a lack of research about experimental fieldwork, that is, studies addressing how students and teachers are trained for individual work phases within the framework of understanding the concepts and the preparation and realisation of experimental work, as well as analysis of the results. One study (Seanpuk & Ruangsuwan, 2017) points out, for instance, that most preservice teachers are hindered by a lack of knowledge and understanding, even regarding a phenomenon as simple as moon phases.

The present investigation uses known working methods for this task (Brglez, 2012; Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010), with the objective of verifying their usefulness in practice. Our aim is to study how suitable individual working methods are for different educational levels, which working procedures require some emphasis to achieve better comprehension, and which procedures can be simplified within the framework of desired measurement accuracy to determine the diameter and depth of craters on the Moon.

In the paper, we first introduce the significance of the determination of a crater's diameter-to-depth ratio. Next, we describe the working methodology in more detail, introducing the course of the experimental work and exposing key working procedures that are the topic of investigation. Subsequently, we present the results, which can be grouped into three topical sets: 1) observation time, 2) work with a telescope, and 3) understanding mathematical working methods. We also provide an analysis of the results regarding the understanding of individual procedures, which offers a basis for simplification in order to enable treatment at different educational levels.

## The significance of the determination of the diameter and depth of craters on the Moon

One of the main questions in the study of craters, which are formed by a collision between a projectile and the Moon's surface, is how to determine parameters that describe the crater's characteristics and enable an insight into the conditions of its formation. We are primarily interested in the initial conditions of the impact, as well as the processes during the transformation of the crater into the final shape that can be observed now. Scott (2002) outlined an investigation into the factors that affect the topography of a simple lunar crater (Figure 1). The ability to predict the topographic dimensions of a simple crater have enabled planetary scientists to determine the stratigraphy of the Moon's crust (Dunkin & Heather, 1999).

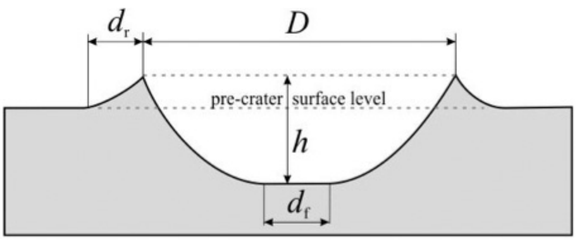



*Figure 1.* Cross section of a simple crater showing the principal features. ($D$ – rim diameter, $d_r$ – rim width, $h$ – depth of crater, $d_f$ – floor diameter)

The two most significant measurable parameters in the description of the crater in Figure 1 are its diameter $D$ and depth $h$, or their ratio. However, there are some limitations of these data in the description of crater formation. When the depth and the diameter are measured, the two large structural parts of the crater are joined: the hollow dug below the initial surface level and the surrounding edge above the surface (Figure 1). In certain studies, such as the determination of the quantity of material dug in the formation of the crater, it is necessary to measure these two structural crater parts separately (Scott, 2013), but this is not so simple with the experimental methods that will be introduced below (Brglez, 2012; Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010), as there is a problem with measurement accuracy. However, if we are only interested in the statistical relationship between the diameter and depth of the craters, our experimental methods are completely adequate. Some of the initial data about the size and depth of craters were, in fact, obtained in the same way. The diameter and depth of numerous craters on the Moon's surface have been calculated based on photographs from terrestrial telescopes and artificial satellites around the Moon, as well as with the knowledge of the angle of sunrays with respect to the Moon's surface (Baldwin, 1965; Pike, 1976; Short & Forman, 1972).

When measuring the diameter and depth of craters on the Moon, it is worth first informing students about the process of crater formation (Scott, 2013; Scott, Shen, Mulley, & Pan, 2013), as they often have misconceptions about the relationship between the size of the projectile and the crater.

The size of the projectile and the crater are similar only in the starting phase of crater formation, when the projectile collides with the Moon's surface: this is the so-called compression phase, as the material in the neighbourhood of the impact location is compressed. This phase depends directly on the momentum and kinetic energy of the projectile. The typical value of the projectile speed on impact with the Moon's surface is between 13 km/s and 18 km/s. After the compression phase, the projectile plays a minor role in the consequent digging of material and the transformation of the crater into the final shape. Part of the projectile and its material evaporates near the impact point due to the high speed and the kinetic energy on impact, and the expansion of the resulting hot gas causes an explosion that distributes the material. The shock wave causes a digging of material, so that a crater forms with a diameter a few times larger than the diameter of the projectile. In the next phase, the compressed material expands again, leading to a small rise of the crater centre, while part of the material from the gravitationally unstable edge and wall slips towards the centre of the crater. During this phase, the crater transforms from a semi-spherical to bowl-like shape, where the power law usually holds for the relationship between the crater's diameter and depth (Scott, 2002).

## Methodology

Our aim was to establish the suitability of the individual phases of the experimental work to determine the diameter and depth of a crater for use in the area of education. We therefore investigated the work of students aged between 21 and 24 who were attending the optional subject of Astronomy in the third year of the first-cycle study programme The Primary School Teacher at the Faculty of Education, University of Maribor. The students serve as a representative sample from the hypothetical population. A total of $N = 47$ students participated in the research. They had no particular knowledge about handling optical equipment and searching for objects in the sky; it was a group of future teachers with middle-school knowledge of natural sciences and mathematical content. In the first year of study, they had attended a refresher course in physics and mathematics, without any specific astronomical topics. The students enrolled in the optional course in astronomy voluntarily based on their individual interest in the subject. The research sample included 47 students of the 2015/2016 and 2016/2017 academic years. They were not taught about the topic of the present research within the framework of the astronomy lectures. The group of students (combining two generations) was heterogeneous regarding their grades in science and mathematics subjects; however, it should be emphasised that, irrespective of their grades, all of the students enrolled in the course due their interest.

The results were obtained on the basis of solving tests, a questionnaire and an evaluation paper concerning our observation of the students' individual work phases, according to the three investigation topic sets mentioned above.

In the first part of the investigation, we were interested in whether the students could accurately estimate the appropriate time to observe Moon craters. Based on the students' work in the computer laboratory, we investigated their ability to search for data about the rising, setting and illumination of the Moon from a particular observation location. This phase is important for planning the time of observation, as the location of the Moon in the sky and its variable illumination influence observation conditions, particularly with regard to determining the depth of craters. In the second part of the investigation, we observed the students' work with a telescope, recording our findings about their skills. The objective of this part was to study how to motivate students for autonomous work when observing with a telescope. In the third part, which involved solving tests and a questionnaire, we investigated the students' understanding of the mathematical working methods that are required to determine the diameter and the depth of the Moon's craters from photographs.

## Results

Below, we present the results regarding how the students understand and are able to perform the different phases of the task of determining the diameter and depth of craters on the Moon. The results concerning the estimation of the appropriate time for observations are given first, followed by the students' success in handling a telescope and their understanding of the mathematical relationships for the calculation of a crater's diameter and depth.

### Determination of the observation time

The first phase of the task, conducted in the classroom, was dedicated to preparation for observation. In the limited time available, the task was for students to use the Internet to find the appropriate time for observing the Moon. We were interested firstly in the extent to which the students were able to find data about Moon visibility at a particular time, and secondly in their choice of the appropriate time to observe Moon craters. We should mention that all of the students included were skilled in searching for information on the Internet. Nevertheless, in the first phase of work, we analysed their skills in finding the specific information required for Moon observations. The time for this task was limited to 15 minutes, so that we could exclude students who had problems with target-oriented searching for information.

***Visibility of the Moon at a particular time***

As shown in Figure 2, 94% of the students successfully found the time periods of the phases of the Moon in the current month. It should be mentioned that they mainly used the web page *http://www.lunin.net/koledar/luninemene/*; although this is an astrological page, it nevertheless contains correct time periods of the phases of the Moon. As expected, the students had more trouble determining the appearance or illumination of the Moon's surface on a particular day (62%), as well as the determination of the rising and setting time of the Moon from a particular observation location (58%) (Figure 2). For the latter task, they mainly used the web page *http://vesolje.net/koledar/koledar.php*, where they could choose between some larger cities in Slovenia.

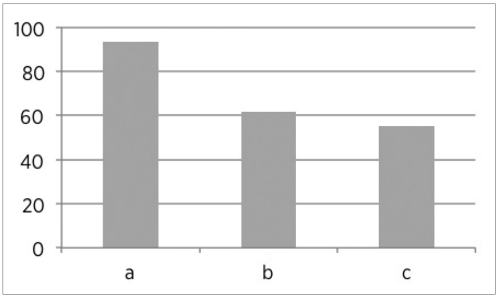



*Figure 2.* Visibility of the Moon.

The share of students who solved the specific task successfully: a) time periods of the phases of the Moon, b) illumination of the Moon's surface on a particular day, c) rising and setting time of the Moon from a particular observation location. The number of students included in the research was $N = 47$.

We found that the students needed some guidance regarding the web pages at which they could obtain data about the visibility of different objects in the sky. On the basis of the analysis of applications and web pages, we found that the most appropriate web page for the students was *http://www.heavens-above.com*, which offers the possibility of choosing any observation location on Earth, while also containing a large amount of data about various objects in the sky. There is also a mobile version of this application available, which facilitates fieldwork.

In our case, we can read the position of the Moon in the sky, its distance, the percentage of illumination, and the rising and setting times, as well as the time periods of its phases from a precisely defined position on Earth.

***Appropriate time for observation***

We were also interested in whether the students were able to choose the appropriate time for the observation of Moon craters. The answer is not uniquely defined, but nevertheless there are some recommendations, as presented below.

Figure 3 shows the students' results regarding the suitability of the moon phases for observation of the size and depth of its craters.

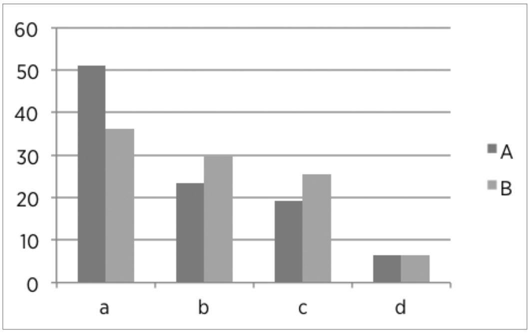



*Figure 3*. The students' opinions (expressed as a percentage) regarding the appropriate time for observation of the size (A) and depth (B) of craters on the Moon: a) full moon, b) first quarter, c) last quarter, d) independently of moon phases. The number of students included in the research was $N = 47$.

It is evident from the results that the students mainly chose the time of the full moon for both of the craters' dimensions. Their most common stated reason for this decision was that the entire visible part of the Moon (the side facing the Earth) is illuminated during this period. A slightly smaller share of the students chose the full moon for observation of the depth of craters (as opposed to observation of their diameter) because they were already aware that a good resolution of the craters' shadows is necessary in order to determine their depth. This reason was stated by students who chose answer *b* or *c*. It is, however, instructive that there is no significant difference in the frequencies of choosing answers *b* and *c*, which suggests that the students are unaware of the fact that the last quarter is observable only in the morning hours, which is not a good time for working with primary school students.

Based on these investigation results, we can conclude that it is necessary to instruct students regarding when each particular phase of the Moon is observable, and to stress the visibility of the craters' shadows in particular relative positions of the Sun, Earth and Moon. We inform students about the advantage of observing the Moon in the period of the first quarter again when the working methods for the determination of the craters' depth are explained.

## Handling a telescope

The fieldwork with the students was undertaken within the framework of field exercises included in the subject Astronomy. This was the first observation work for the students within this subject. A reflective telescope MEADE ACF-SC 203/2000 8" LX200 (Astroshop.eu, 2017) with a focal length of 2 m was used. First, we observed the Moon through the eyepiece with various focal lengths, so that the students could get used to handling the telescope and acquire a feeling for magnification. Photos were then taken with the camera Imaging Source DFK 41AU02.AS Astro (Astroshop.eu, 2017).

We observed the students' work and wrote down our findings. We were interested in the following skills:

a) setting the sharpness of the image of the observed object;
b) adapting the telescope position in order to always keep the Moon in sight;
c) the correct choice of eyepiece according to the desired magnitude;
d) taking photos with the CCD camera.

For more than 70% of the students, this was their first experience of observing the sky through a telescope. Even those with previous experience had never handled a telescope by themselves. We therefore had to introduce the individual working methods and procedures to the students accordingly.

Despite these difficulties, the students quickly learned to handle the telescope. Setting the sharpness of the image (a) presented a minor obstacle for them, and some training was needed to acquire the skill of adapting the telescope position (b). However, the most problematic aspect was choosing the right eyepiece (c), with the students initially choosing among available eyepieces quite randomly. It was clear that they needed some additional explanation about the choice of the eyepiece, as well.

If we want to catch the whole Moon in the telescope lens, we have to choose a visual angle of 0.5°. When using the telescope LX200 with a focal length of 2000 mm, this visual angle is achieved with an eyepiece with a focal length of 20 mm and a visual angle of 52° (TS-Optics Super-Plözl, Series 4000, 52°). Here, the magnification of the telescope is 100 and the corresponding visual angle is 52°/100 = 0.52°.

## Understanding the working methods

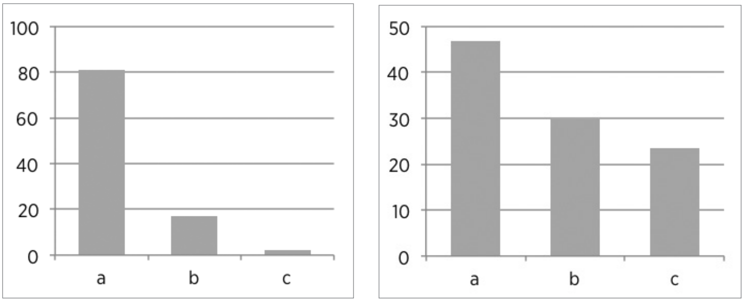
The focus was on commonly known methods related to determining the diameter and depth of craters on the Moon (Brglez, 2012; Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010), our objective being to establish the extent to which the students understood these methods. In order to determine the scale ratios of the objects on the photos, we needed digital photos of the Moon and a computer programme. We used the commonly known programme Microsoft Paint, which is among the Accessories of the Windows operating system.

***Diameter of craters***

If we want to calculate a crater's diameter, we use a simple calculation of the ratios of the known and unknown lengths. The procedure, which is described in detail on the web page of Martin Brglez (2012), was introduced to the students. We investigated the extent to which the students understood the procedure, as well as their awareness of the limitations and measurement errors connected with this task.

Figure 4A shows that the students had no problems with calculating the ratios in question (eq. 1), although it is somewhat surprising that 19% of them did not understand the method completely, despite the fact that ratios of this kind are treated thoroughly as early as in primary school.

As expected, the results regarding understanding the limitations of the aforementioned method were worse (Figure 4B). The method (eq. 1) gives the exact value only for craters that are in the centre of the circle in Figure 5, when the surface of the Moon is perpendicular to the observation direction. For craters outside this area, it is necessary to consider the projections of the inclination of the Moon's surface relative to the observation direction. It is obvious from figure 4B that more than half of the students were unaware of this problem.




A B

*Figure 4.* Percentage of students who understood the method for calculation of the diameter of craters on the Moon: A) Understanding of Ratios: a) complete, b) partial, c) not at all; B) Understanding the Limitations of the Method: a) it works for craters in the centre of the outlined circle; b) it works for craters at the terminator, c) it works for all craters. The number of students included in the research was $N = 47$.

As an example, we take a crater named Ptolemaeus (Figure 5), which has a diameter of 154 km (Legrand & Chevally, 2012). Taking the Moon diameter as $2r$ = 3,474 km, the students determined the following crater diameter (on average):

$$x = 2r\ \frac{x'}{2r'} = 152{,}2\ \text{km}, \qquad (1)$$

where $\frac{x'}{2r'}$ is the ratio of the number of pixels on the screen.

We conclude that the relative measurement error in the students' work was about 2%.

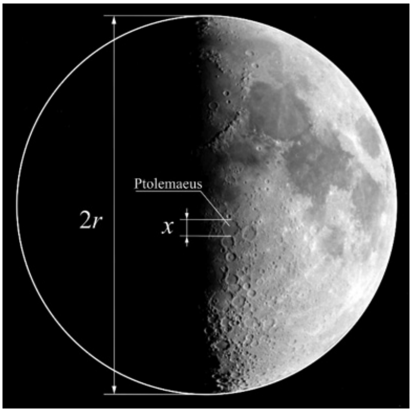



*Figure 5.* Determination of the diameter of a chosen Moon crater.

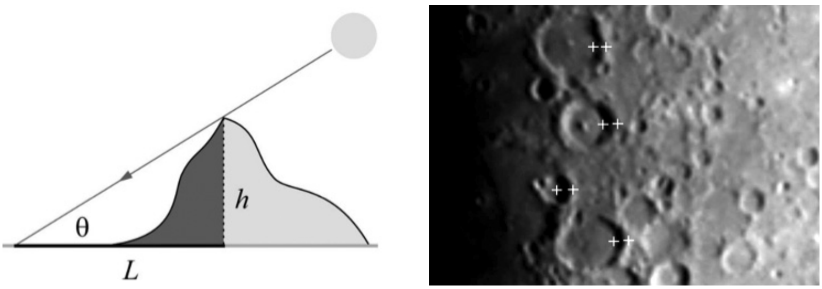
***Depth of craters***

Regarding this task, a fairly accurate method was presented to the students, whereby the depth of craters or the heights of mountains on the Moon can be measured within an error of 10%. This method is described in more detail in the Slovenian astronomic review *Kmica* (Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010). The method for mountain height $h$ is based on the length of mountain shadow $L$ (Figure 6A). The following relationship holds:

$$h = L\ \tan\theta. \qquad (2)$$

Here, it is necessary to measure the shadow length ($L$) as accurately as possible, and to determine the angle ($\theta$) between the sunrays and the Moon's surface.

The length of the shadow is determined in a similar way to the determination of the crater diameter, but we need to magnify the photo appropriately and precisely determine the top (bottom) of the mountain (crater) and the edge of the shadow (Figure 6B). In this way, we determine the virtual shadow length, not the true length. In order to determine the true length of the shadow, we need to include the corresponding corrections due to the spherical shape of the Moon (Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010). We will not explain these corrections here, as they were very difficult for the students to understand (Figure 7A). Furthermore, the corrections can be neglected, at least if we analyse craters near the point on the line through the observer on the Earth and the centre of the Moon. The error at craters lying within the apparent circle with a diameter one half of the apparent diameter of the Moon around the aforementioned point is roughly less than 15%. At larger distances from this point, the error increases rapidly.




A B

*Figure 6.* Determination of the height/depth of a mountain/crater: A) Ratio between the shadow length and the height; B) Determination of the shadow length on the photograph.

Finally, we also have to know the angle between the sunrays and the Moon's surface. There is a detailed explanation of this angle in the reference material (Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010). Knowledge about spherical geometry is required in order to understand the mathematics of this problem; however, the majority of the students lacked such knowledge (Figure 7B).

With regard to this problem, one option is that we simply assume the correctness of the final equation:

$$\theta = \arcsin\left(\sin\varphi \cdot \sin\varphi_s + \cos\varphi \cdot \cos\varphi_s \cos(\lambda_s - \lambda)\right) \qquad (3)$$

Here, the pair of parameters ($\lambda$, $\varphi$) determines the position of the crater on the Moon, while the pair ($\lambda_s$, $\varphi_s$) gives the position of the so-called "subsolar point", which is the point on the Moon where the Sun is at its zenith. We can find the crater position with the programme Virtual Moon Atlas (Legrand & Chevally, 2012), while the subsolar point can be found on the web page *http://www.lunar-occultations.com/rlo/ephemeris.htm* (Burnett, 2000), where we simply enter the time of observation. For instance, on the day of our experiments, 3 April 2017 at 21:00, the corresponding parameters in angular degrees were $\varphi_s = 1^0 S$ and $\lambda_s = 93{,}2^0 W$.

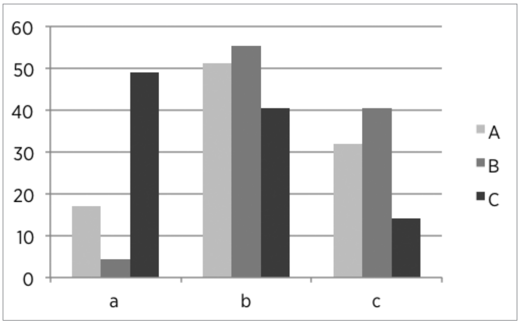



*Figure 7.* Percentage of the students with an understanding of the method for determining the depth of craters on the Moon: A) Understanding corrections of the length, B) Understanding spherical geometry for calculation of the position of the Sun above the horizon, C) Understanding the method with plane geometry; a) complete understanding, b) partial understanding, **c)** no understanding at all. The number of students included in the research was $N = 47$.

Since understanding the method in the reference material (Kelemen, Šomen, Bohinec, Davidović, & Gomboc, 2010) is too demanding for students, we suggest a simplified method, whereby the position of the Sun above the horizon is calculated within plane trigonometry (Figure 8), which is treated in secondary school. A better understanding of the simplified method is evident from the comparison of Figures 7B and 7C. In this way, the method turns out to be appropriate for teaching in secondary schools, as well.

In this case, the height of the sun is given as the difference of the angles (Figure 8A):

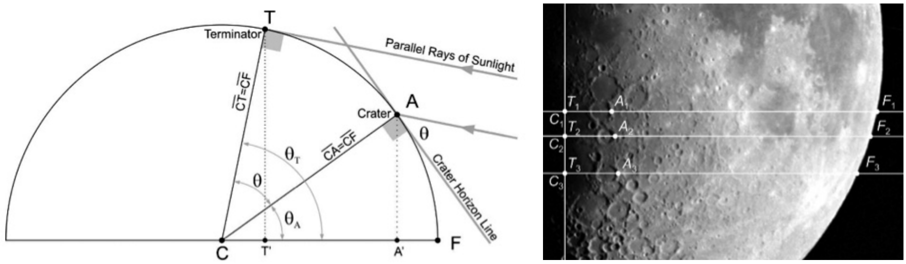
$$\theta = \theta_T - \theta_A \tag{4}$$

The angles $\theta_T$ and $\theta_A$ are calculated from the ratios of distances:

$$\cos \theta_T = \frac{\overline{CT'}}{\overline{CF}} \tag{5 a}$$

$$\cos \theta_A = \frac{\overline{CA'}}{\overline{CF}} \qquad (5\,b)$$

Using this simplification, the students measured the depth of craters within an error of 10%.




A B

*Figure 8.* Simplified method for determining the depth of craters on the Moon: A) Geometry with angles and distances in eqs. (4) and (5); B) Reading the ratio $\overline{CA'}/\overline{CF}$ in Eq. (5b).

If we are working with younger students, such as primary school students, trigonometry is either beyond their scope or, at best, known only on a basic level. The procedure in Figure 8 using equations 4, 5a and 5b can be performed through construction with a ruler and protractor instead of through calculation. This is of great importance for teaching younger students. This hands-on approach would also aid in understanding the concept of projection and angles much better than (only) inserting numbers into equations, which most students see as an abstract process, separated from easily imaginable geometric reality. With this approach, school students acquire the same physics in the easiest possible way, without difficult mathematical obstacles.

It should be emphasised that this method for determining the angle $\theta$ is correct for the longest value of $\overline{CF}$ in Figure 8B, where the plane, determined by the direction of sunrays and $\overline{CA}$ (Figure 8A), is perpendicular to the tangential plane of the Moon's surface at point A. For a shorter $\overline{CF}$ this method does not take into account the angle between the tangential plane of the Moon's surface at point A and the plane through point A, perpendicular to $\overline{CA}$ (Figure 8A). This error is more significant for craters near the terminator. We estimate that the value of this error does not exceed 15%, where $\overline{CF}$ is one half of the maximal $\overline{CA}$.

We did not systematically ask the students about their opinion of the topic and method of teaching (and possible suggestions for future work). However, we observed a high level of student motivation during all phases of learning. In addition, the present teachers confirmed the positive impact of this course on the working atmosphere in the classes.

## Conclusions

In education, the quality of the conditions for experimental work within the framework of astronomy topics is improving due to the enhanced interest of students and to projects that enable the financing of astronomic equipment (Vičar, 2009). In our case, the topic is the observation of craters on the Moon. We have shown that observation with a telescope can be upgraded with methods that enable the measurement of the diameter and depth of craters. Based on an analysis of the results of students' experimental work by individual phases, we found that experimental work involving the observation of craters on the Moon can be incorporated into physics or astronomy lessons at different levels of education by: i) introducing suggestions concerning searching for information, ii) using some crucial focuses in explanations, and iii) possibly applying appropriate simplifications to some otherwise demanding methods.

We found that students need some guidance regarding web pages on which they can find data about the visibility of various objects in the sky. On the basis of an analysis of applications and web pages, we found that the most appropriate web page for students was *http://www.heavens-above.com*, which offers the possibility of choosing any observation location on Earth and contains a large amount of data about various objects in the sky. In our case, we could use the website to find out the position of the Moon in the sky, its distance, the percentage of illumination, and the rising and setting times, as well as the time periods of its phases from a precisely defined position on Earth.

With regard to handling the telescope, the students did not have any major difficulties. The only explanation required concerned the choice of eyepiece for the desired visual angle of the Moon. The students can easily take photographs with adequate equipment. We should note that, when it is not possible to take photos, the students can use photos available on the Internet. However, they should take care that such images are comparable with images seen through a telescope lens. An adequate similarity of the image seen through telescope and the image from Internet must be achieved for the same moon phase and with clear visibility of the shadows of the craters. The resolution of the image from the Internet must be at least of the same quality as the image seen through the telescope, or better. In the case of better resolution, the students can investigate the influence of the resolution on the accuracy of the measurements. However, making photos of the Moon and other astronomic objects is in general very motivating for students. This is an opportunity to incorporate this kind of experimental work into the students' other research work, such as measuring the apparent size of the Moon (Ellery & Hughes, 2012) and determining the size of the Sun (Guglielmino, Gratton, & Oss, 2010).

When calculating a crater's diameter, we use a simple calculation of the ratios of known and unknown lengths, which uses mathematics already acquired in primary school. However, it is necessary to warn the students that this method only works for craters where the Moon's surface is perpendicular to the observation direction. This requirement can be explained in more detail at a higher level of education. Similar findings hold for the estimation of the depth of craters (or the heights of mountains), where it is necessary to measure the lengths of the object's shadow as accurately as possible, and to determine the angle between the sunrays and the Moon's surface. We found that the method involving spherical geometry is too demanding even for most high school students. Thus, we suggest a simplified method with planar geometry, which students understand much better. In addition, this method gives measurement results with comparable accuracy to the more demanding method with spherical geometry when certain limitations are taken into account.

We should emphasise that the results of the present research cannot be generalised due to the specific properties of our sample. In a didactical sense, additional research needs to be done by testing students in primary and high schools. It is also important to test the teachers in schools where the sample is the most similar to our sample. The combined results may yield an optimal recommendation on how to provide experimental work for this topic in schools.

The method used for determining the depth of craters on the Moon can be upgraded with experimental work in class, whereby we can use the same method to determine the depth of craters formed by the impact of objects with soft ground (Scott, Shen, Mulley, & Pan, 2013). In a darkened classroom, we can apply different angles of light illumination to these craters. Furthermore, we can study the influence of the size and kinetic energy of the projectile, as well as the influence of the density and structure of the ground on the crater's size. Of course, it is necessary to inform the students about the main differences between the formation of craters in a school experiment and in the natural environment. The size of the crater in class is comparable with the size of the projectile, while the size of a naturally formed crater is several times larger than the projectile, as mentioned in section two of the present paper.

Based on our findings, we can conclude that, with some adaptations, experimental work such as that described in the present paper is appropriate for different educational levels, and that it enables students to gain specific natural science and mathematical competences that are also required for the study of other natural phenomena. It is worth stressing the generic competences directly linked with the experimental work; for instance, the ability to collect and analyse information, and the ability to reach conclusions.

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