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Mathematics Learning from an Interactionist Perspective

Abstract

The section presents an attempt to develop a theory of mathematics learning based on social interactionism. It takes into account the specific features of the academic domains of the mathematics subjects to be learned. Three basic assumptions are outlined in detail. (1) Learning is situationally bound in an interactional process of *cooperation* between the participants of a situation based on the negotiation of meaning. (2) The indicator of a successful process of learning is an increased autonomous *participation* in such cooperative interaction. This encompasses the *acquisition* of mathematical concepts and procedures, as well as the specific *reasoning* in mathematical teaching-learning discourses. (3) The constitutive social condition of the possibility of learning mathematics is the participation in *collective argumentations* that refer to mathematics-related terms and procedures. These domain specific kinds of discourse intertwine the two traits of learning – acquisition and reasoning – under a common perspective.

Keywords: social interactionism; negotiation of meaning; social participation; collective argumentation

> Despina can be reached in two ways: by ship or by camel.
> The city displays one face to the traveler arriving overland and
> a different one to him who arrives by sea.
> (Calvlino 1972; Cities and Desire 3)

1 Introduction: the theoretical framework of an interactional theory of mathematics learning

My aim is to outline contours of a theory of mathematics learning that incorporates the specific subjects of the academic domain of mathematics to be learned and that is construed on the basic assumption of social interactionism. From the perspective of mathematics education, psychology seems to be the predominant science that is concerned with issues of learning mathematics.

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As a result, psychologically based research projects on aspects of learning mathematics can be found in the discipline of mathematics education. But if a mathematics education researcher looks at the mathematics that is the subject of the psychology of learning, it becomes clear that the theoretical interests are general statements that do not take into account the specificities of the academic domain of mathematics. In psychology, mathematics often stands for any content-related domain of knowledge and is utilized for research purposes primarily because of its seemingly clearly structured content and its easy testability. One seldom finds projects in this field that reflect the specific features of the domain of mathematics in a way that one would expect in the community of mathematics education researchers (see Krummheuer 2013). However, the theory of mathematics learning that I am focusing on is not seen as an application of such a psychological theory of learning. The theory to be developed here is a theory of mathematics learning that constitutively respects the specific features of the academic domain of the subjects to be learned (Krummheuer, 2014).
In addition to this focus on the domain of mathematics, the perspective on the development of such a theory of mathematical learning is one of social interactionism. It is employed in order to establish a theoretical stance that is not characterized by the universally psychological view on the cognitive development of the single individual. Thus, we call it an *interactional theory of mathematics learning*. Mainly, this approach refers to the phenomenological sociology of Schütz & Luckmann (1979) and its expansion into ethnomethodology (Garfinkel, 1972) and symbolic interactionism (Blumer, 1969).
There seems to be a kind of chasm between a psychological concept of learning and a sociological one that inhibits the development of a unifying theory. Therefore, Cobb and Bauersfeld (1995) speak of a *coordination* of two different perspectives rather than of the possibility of generating an overarching approach that encompasses both perspectives. They conclude with regard to this seeming chasm:

> "This coordination does not, however, produce a seamless theoretical framework. Instead, the resulting orientation is analogous to Heisenberg's uncertainty principle. When the focus is on the individual, the social fades into the background, and vice versa." (p. 8)

This quote might be seen as a scientific description of the phenomenon that Calvino mentions in the epigram above as a traveling poet, if one replaces "Despina" by the "learning" and "ship" and "camel" by "sociology "and "psychology".[1] Somehow the dilemma of uncertainty defines limitations that obviously

1 I leave it to the reader, how he*she assigns the ship and the camel to the sciences of sociology and psychology.

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cannot be transcended as in the different views of Despina depending on how one approaches this city.

In the field of mathematics education, often one characterizes a sociological based theory of learning as the Vygotskian perspective (Wertsch & Tulviste, 1992). Specifically, I refer to Max Miller's (1986) approach of a sociological learning theory. He states that the individual's learning is created by the attempts of the members of a group to collectively clarify their interindividual problems of coordinating their actions. For Miller there exists only one communicative type of action that can successfully solve these attempts, and that is the *collective argumentation*.

> "One can assume that only such social or communicative actions can provoke fundamental learning processes, which primary goal and which functioning stand for developing collective solutions for the interindividual problems of coordination. There is only one social or communicative type of action that fulfills this condition, and this is the … collective argumentation." (p. 23).

This still sounds very much like features of rather idealized unconstrained interaction, in which all members participate voluntarily, without any hesitancy of expressing his*her point of view, and without being interrupted or distracted by interventions of other participants (Habermas, 1985). In this approach, the concept of collective argumentation is not to be seen as an empirical description of teaching learning situations but according to Blumer (1954) rather functions as a "sensitizing concept" (p. 7; see also Blumer, 1969). These kinds of concepts help to develop a theoretical perspective that describes new aspects, points of reference and basic assumptions. They build the necessary theoretical skeleton, which one has to complement with definitive and empirically grounded concepts.

I use this concept of collective argumentation as a sensitizing notion, taking it as a discourse that is coined using explanations and justifications for the mathematically related actions at stake. In the following, I enrich this concept by reshaping it by means of several rather empirically grounded concepts that allow me to elaborate the inclusion of the mathematical domain specificities in this theoretical approach.

Referring to Miller (1986), the theoretical considerations that are to be delineated in the following are based on three basic assumptions:

1. Learning is necessarily situationally bound in an interactional process of *cooperation* between the participants of a situation based on the negotiation of meaning.
2. The indicator of a successful process of learning is the increased autonomous *participation* in such cooperative situations of interaction. This incremental

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process toward "full participation" (Lave & Wenger, 1991, p. 37) is facilitated by the development of individual competence, which encompasses the *acquisition* of mathematical concepts and procedures as well as the specific *reasoning* in mathematical teaching-learning discourses.

3. The constitutive social condition of the possibility of learning mathematics is the participation in a *collective argumentation* that refers to mathematics-related terms and procedures. The domain specific kind of discourse intertwines the two traits of learning – acquisition and reasoning – under a common perspective.

In the following I refer to these three topics in more detail.[2]

2 Cooperation – the coordination of action within a process of collective argumentation

Below I discuss two aspects of everyday interaction, which widen our understanding of a sociological view on learning.

- First, the mentioned "collective solutions for the interindividual problems of coordination" (Miller, 1986, p. 23) are interactive endeavors, which emerge in the interactional exchange among several participants. The participants have to find means and methods for coordinating their contributions. This course of interaction is embedded in the all-embracing process of *negotiation of meaning*.
- Second, negotiation of meaning can proceed in routinized forms and therefore one has to consider how to adapt the sensitizing concept of collective argumentation to such patterned interactions.

2.1 Negotiation

As outlined, the original idea of a sociological conceptualizing of learning is that the participants in an interaction situation coordinate their different opinions about a developing theme by means of an interactional exchange. Finally, this leads to mutual interpretations of the situation that is based on the most convincing argument. This takes place within the all-embracing process of negotiating meaning: in order to act together, the participants of an encounter have to adjust their momentary interpretations of the particular situation, and this happens by negotiation. With respect to a teaching-learning situation in classrooms, I conceptualize this process of negotiation as the progression of coordinating the different perspectives of a given mathematical theme: the

2 The following remarks refer in larger parts to Krummheuer (2023).

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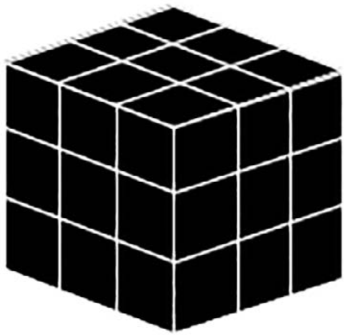
teacher usually frames this situation rather in terms of his*her advanced mathematical expertise whereas the students still are unable to do so and they therefore interpret the same event in different ways.

On a theoretical level, I mold this divergence of meaning-making with regard to an emerging mathematical theme by employing Goffman's (1974) concept of *frame* and *framing*, and describe this divergence as a difference in framing. Hereby, frame is a routinized and standardized configuration of a definition of situation. *Framing* is the process of conducting an interpretation by activating a frame. Principally, a frame is a concept that refers to the individual achievement of assigning meaning to ongoing activities. However, through previous negotiations of meaning in similar situations, these frames usually manifest the common-sense interpretation of a certain social group. Hereby, my main interest refers to the common-sense interpretations of mathematical subjects in classroom interactions within the process of an emerging process of collective argumentation.

To illustrate consider the following math-problem for primary school children (see Figure 1; Radatz & Rickmeyer, 1991, p. 77ff.):



Imagine, you have a large cube made of light wood. You would paint it entirely black and then saw it apart, as the picture shows. Question: How many cubes would have three black sides?

Figure 1: Cube divided into smaller cubes.

In one of my studies (Krummheuer, 1997), two second graders framed the picture as a two-dimensional geometric pattern and then tried to figure out how many rhombi are contained in the picture. In this framing, the whole *story* in the given task of painting a cube appears to be of minor relevance. They might have framed the problem in the sense of a school-geometrical frame as used in *similar* problems like 'How many cells are in this rectangle?' (see Figure 2)

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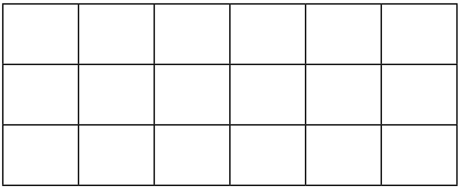

Figure 2: Two-dimensional geometric pattern

Taking into account that this task is part of second grade mathematics, the children's framing might include the interpretation of this rectangle as a visualization of a multiplication: 5 cells in a row times 3 cells in a column equals 15 cells in total.
In contrast, the teacher framed the picture as the projection of a cube in a two-dimensional plane and could therefore also speak of hidden sides of the original three-dimensional object. In the conversation between the teacher and the children arose a framing-conflict. Incidentally, a successful collective argumentation emerged, when the teacher showed them a concrete wooden cube. Thus, the initially accomplished framing conflict could be mitigated allowing the interaction to proceed. Needless to say, this final conversation does not include the original problem of painting and sawing a cube.
From a mathematical stance, both interpretations of the *graphic* are valid. There is no wrong or right. Therefore, the differences of the framings can only be transcended in a collective argumentation that is based on an alternative approach of interpretation. In the concrete case, this happened by adding a concrete wooden cube into the interaction, which engendered a common view on a three-dimensional object.

2.2 Interaction pattern

Looking at mathematics classroom interaction we often can reconstruct interaction processes that emerge along the steps of an interaction pattern, although we can assume a difference in the interpretation of the situation by teacher and students. In such cases the coordination of the situation hardly ever appears explicitly, but still one has to consider that these are interaction processes in which students are enabled to learn mathematics. That means, that a collective argumentation occurs as explained in the following.
The approach of ethnomethodology is beneficial, here. According to it, we differentiate between the *performance* of actions and their *accountability*. In terms of the original idea of collective argumentation we would state, that after participants can debate explicitly their different point of views, a discourse of an explicit argumentative exchange emerges about its accountability. Looking

at the case of patterned interactions, in which seemingly no dispute arises and the actions for the participants seem to appear sufficiently "self-explanatory", we can state in the words of Garfinkel (1967, p. 280) a specific *accounting practice*, in which:

> "the activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings 'account-able'" (p. 280).

Let me present three examples of such accounting practices.
The first example is a serious conversation among people. A person wants to make clear, that that what he*she is going to say is absolutely serious. The way that this person makes his*her utterance shows concurrently, that he*she is absolutely accountable for that what he*she is saying.
The second example is just the opposite of a serious conversation, namely a joke. Usually, the person who tells a joke does not have to explicitly pronounce that he*she is telling a joke. Rather, in the way he*she speaks for all participants, it is apparent that this is a joke. Thus, the action of telling a joke is identical with the procedure of making such a situation funny.
The third example is the teacher's introduction of a new mathematical subject matter in a regular mathematics lesson. Usually this happens by presenting a mathematical problem for which the solution is still unknown for the students. The teacher tries to present this new matter in such a way, that he*she expects that his*her students can follow his*her thematic unfolding and can accept the concluded mathematical solution. In a scientific domain like mathematics the presentation of the teacher is based on the accomplishment of an argumentation, which is supposed to convince the students of the correctness of the given demonstration.
By ethnomethodologists, these examples demonstrate an *accounting practice.* It relates the performance of one's action *reflexively* to the kind of accountability this person wants to assign to his*her action. Of course, in all three examples this reflexivity can fail. Then one has to explicate the reasoning of the requested accounting practice. In the first example, one could say: 'I am absolutely serious about this' in the second case, one could say: 'well it's just a joke'. And in the example from the maths class, the teacher could say: 'I guess, I should explain my thinking once more in a different way'.
That means, accounting practices can either emerge *discursively*, when performance and accountability are thematized separately, or *reflexively*, when performance and accountability coalesce in the flow of the interaction. Accordingly, we speak of a "discursive accounting practice" or a "reflexive accounting practice" (Krummheuer & Fetzer, 2005, p. 30).

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In everyday mathematics classroom situations, we can reconstruct patterns of interaction characterized by a reflexive accounting practice that, through its execution, adheres to collective argumentation.

3 Learning

Here I address the question in which way the interactional process of negotiation of meaning is related to the individual process of learning mathematics. As mentioned above, Cobb and Bauersfeld (1995) claim a complex relationship between these two perspectives on learning that is similar to the uncertainty relation of Heisenberg. With respect to Miller (see above) the notion of collective argumentation appears as the striking aspect for the coordination of the actions of the participating individuals in a social encounter that relates to the individual process of learning. How does it happen though?

The crucial point is that the relationship between collective argumentation and learning is defined by the conceptualization of learning as an argumentative process. The idea is not that the students have to learn to argue in order to defend their position or to convince others by arguing for it. The idea is rather that the participants are altogether engaged in the accomplishment of an argumentation that is convincing for all of them. One might call it *argumentative learning* in contrast to *learning arguing*. The learning of mathematics in this sense includes two aspects:

- the *acquisition of mathematical content*, like arithmetical procedures or geometrical facts and
- the *development of mathematical thinking* situated in a mathematically framed practice of collective argumentation.

The shibboleth of the realm of argumentative learning is the concept of *participation*. Sfard (2008) characterizes this position as "participationism" (p. 76). She introduces the metaphor of "learning-as-participation" (p. 92) for this viewpoint, which I prefer to alter into *learning as incremental growth of participation.*

Like collective argumentation, participation in a discourse does not by itself represent a learning process. Learning takes place, if the student in question is

- incrementally enlarging his*her share of participation and by this
- proposing qualitatively more sophisticated contributions

to the interactive accomplishment of a mathematically framed collective argumentation.

Roughly, Lave and Wenger (1991, p. 37) explain such a process, whereby they do not focus neither on collective argumentation nor on mathematics learn-

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ing. They describe it as the process from taking over a "legitimate peripheral participation" in the beginning and moving to the role of a "full participant" (p. 37) in the end of the learning process. Krummheuer and Brandt (2001) developed a more sophisticated model of participation in classroom situations that refers to a socio-linguistic approach from Goffman. He elaborates the notion of a "recipient design" for the different kinds of listening and of a "production format" for the various forms of actively generating an utterance in an interaction with others (Goffman, 1981). Based on his work one can design a potential trajectory of incrementally growing phases of participation that reflects a learning process of a student. It starts with the

1. legitimate peripheral participation in the sense of listening, develops into a
2. participation role that is characterized by repeating or reiterating someone else's utterance, then goes on to
3. paraphrasing a mathematical idea, that has been previously introduced by someone else and by putting this idea in one's own words, then proceeding by
4. expressing one's own new mathematical ideas, still by referring to formulations that came from other participants, and to finally conclude by
5. introducing new original ideas in own authentic formulations like a competent author, which represents a full participation.

In the last point, I use the concept of the 'competent author'. His*her competence is characterized by the correct application of mathematical concepts and procedures, and additionally, by appropriately contributing to the emerging process of a collective argumentation. According to this approach, one can characterize the interaction process of collective argumentation as a specific discourse that functions as a *Mathematics Learning Support System* (MLSS) in which the students gradually advance in their participation roles.

Here I refer to Bruner's (1982) work about young children's acquisition of their mother tongue. According to him a young child does not acquire its mother tongue in the sense of "cracking a linguistic code" (p. 14) but rather in the sense of adopting to the "demands of the culture" (p. 103). Learning is facilitated by involving the learning individual in an interactional support system in which the more competent members care that the learning child can take part more autonomously in the process of the incremental growth of participation. In terms of learning mathematics, it takes place by accomplishing processes of collective argumentations.

Schütte et al. (2021) integrate and refine this notion of MLSS in their theory of mathematics learning. They reconstruct in the small group interaction between a nurse and several three-year-old children in preschool and kindergarten different styles of argumentative discourses, which they call "formal",

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"narratory" and "narrative" (Schütte & Krummheuer, 2013; Krummheuer, 2016; Schütte et al., 2021). Without going into detail here, they are able to show that the kind of participation in these discourses of collective argumentation differs for each child. For example, one girl was able to participate actively and autonomously in a narrative discourse and the evolving MLSS was supportive for her. For another boy it was just the opposite, he participated more actively in a formal discourse. They conclude that MLSSs are not equally supportive for each child. They refer to support systems that enable children to improve their participation statuses as profitably as possible as the "interactional niche for the development of mathematical thinking" (NMT). They adopt the concept of "developmental niche" from Super and Harkness (1986).

4 Argumentation

In this section I elaborate the concept of collective argumentation. As mentioned above, this concept encompasses the character of a sensitizing concept according to Blumer. These elaborations are empirically grounded ideas. Blumer calls them *definite concepts* (Blumer, 1954, p. 7). By speaking of *empirically grounded* ideas, I additionally refer to the methods of qualitative research as they are compiled under the wording of *grounded theory* (e.g., Glaser &Strauss, 1967, Strauss & Corbin, 1990).

Coming back to my concern of clarifying the concept of collective argumentation in more detail, I first emphasize that the analysis of argumentation in a classroom should not misleadingly be understood as a treatise on proof (Krummheuer, 1995). Both the concepts of argument and argumentation need not be exclusively connected with formal logic as we know it from mathematical proofs. There are more human activities and human efforts that are rational and based on argumentative demonstrations. As Toulmin (1969) points out, if these formally logical conclusions would be the only legitimate form of argumentation at all, then rational communication would be extremely restricted. Argumentation would be rather irrelevant as a possible way of communication based on rationality.

Toulmin calls argumentations, which follow the strict rules of a deduction "analytic" (Toulmin, 1969, p. 113). These forms of argumentations contain in their conclusion nothing that is not already a potential part of the premises. In contrast, "substantial" arguments (p. 113) expand the meaning of such propositions insofar as they soundly relate a specific case by actualization, modification and/or application. Thus, substantial argumentations are not necessarily based on the logic of deduction. They rather transfer the verisimilitude of given propositions to the specific case that is under scrutiny. A substantial argument is effective, when, finally, the doubting participants are convinced

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by this argumentative demonstration. An analytic argumentation, in contrast, aims at the logical demonstration that the statement at stake is true, irrespectively whether the listeners can cognitively track these logical deductions or not. The target of a substantial argumentation is to convince and to persuade that the statement at stake finds the assent of all participants (Perelman & Olbrechts-Tyteca, 1969, p. 4).

It seems very helpful to employ this conceptual differentiation for the analysis of processes of argumentation in primary mathematics classroom activities. The character of argumentation in this setting is rather a substantial one. With the differentiation between analytic and substantial, we can apply the approach of argumentation more suitably to our field of interest.

With regard to the concept of collective argumentation, it also proves helpful to differentiate between the *academic task structure* (ATS) and the *social participation structure* (SPS)[3]. Usually, in mathematics classroom interaction, there is a task to be solved including a collective argumentation. Thus, ATS refers to the interactionally accomplished steps of actions. There might be only one individual involved, but usually several students and the teacher jointly produce these solving steps in an interactional exchange about solving the task. This exchange involves SPS. Both are simultaneously interwoven and describe the concrete process of a collective argumentation.

Referring back to the example of the cube, think of two second graders named Esther and Linda. After quickly reading the text of the given problem, a short dialog emerges.

1	Linda	*Points at the picture*[4] three parts, wait *counts the areas of the left side* one, two, three, four, five, six, seven, eight, nine … three times nine
2	Esther	Three, six, nine
3	Linda	twenty-seven
4	Esther	twenty-seven

Later the teacher asked them what they had done. They answered:

10	Esther	*Circles with her index finger over the picture* we counted one and then we multiplied by three
11	Linda	and then we multiplied by three

3 The notions of ATS and SPS refer back to Erickson, F. (1982). Classroom discourse as improvisation. In L. C. Wilkinson (Ed.), *Communicating in the classroom.* (pp. 153 - 181). Academic Press.; see also Krummheuer, G., & Fetzer, M. (2005). *Der Alltag im Mathematikunterricht. Beobachten, Verstehen, Gestalten.* Spektrum Akademischer Verlag., p. 45.

4 Italics describe non-verbal actions.

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In retrospect, Esther presented parts of her accomplished ATS:

1. Count the sides of the object in the picture – the result is 3
2. Count the areas of one of the sides – the result is 9
3. Multiply the two results – 3 x 9 = 27

For this argumentation, it remains implicit that the three "sides" contain the same number of sections. The persuasive power of the accomplished substantial argument for the two girls might reflect

- their experience with the visualization of multiplication in form of a split rectangle in rows and columns (see above),
- the narrative verisimilitude of counting (Krummheuer, 1999).

As a note, while Linda seems to be more dominant during the solving process, it is Esther who explains. Both girls seem to be very convinced of their accomplished argumentation.

With respect to the SPS, one can reconstruct that Linda mentions the three sides of the object and suggests to count the areas of one side. Both girls count, Linda in steps of one and Esther in steps of three. Both children present their result of the multiplication 3 x 9. Thus, the emerging SPS seems to be characterized by a relatively symmetrical participation of the two children.

To summarize, the collective argumentation in this short example leads to an obviously very convincing argument for the two children, while they interact in a relatively symmetrical way. The domain specificity of the accomplished collective argumentation is primarily located in the ATS. One can assume, that for both children the whole situation functions as a MLSS, though, unfortunately not for the geometry of cubes but for the arithmetic of multiplication. In the latter case, we are dealing with NMT.

Some readers might be astonished that I do not further elaborate on the fact that the two children did not solve the given problem in the expected way. Although I am aware of this, as an ethnographer of mathematics classroom interaction, I am obliged to neutrally observe without criticizing the results of the interaction. From a rather normative perspective of mathematics education, one can draw on such research results and develop suggestions on how to organize interaction in mathematics classroom interaction, and furthermore, reflect on what kind of mathematical problems one can use to support such collective argumentation for a satisfying learning effect.

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5 Conclusion

Mathematics learning is situated in processes of negotiation that arise when several individuals try to act together in social encounters dealing with a mathematical theme. The typical constellation of these processes is characterized by framing differences. If this coordination happens on the basis of a rational exchange, then we are dealing with a process of collective argumentation. This is a discourse that functions as a MLSS for those who still have to learn about the themes at stake. An indicator for a successful learning process is the incremental growth of the participation of the learner in the ongoing interaction process. For some of the learners, the emerging support system might be functional. In this case we characterize this specific MLSS as an interactional niche for the development of mathematical thinking (NMT). From the perspective of argumentation, such MLSSs represent a specific accounting practice that emerges either in a discursive or in a reflexive way. This depends on how explicitly the differences in definitions of the situation created by the participants are formulated. The collectively generated argumentation might lead to an analytic or a substantial argument.

According to this theoretical approach, the domain specificity of classroom interaction is included in the ATS of the emerging processes of collective argumentation. Clarifying this position with respect to *mathematics* classroom interaction in primary education, it is not as much the stringency and analytic character of mathematical proofs, it is rather the multitude of substantial argumentations that aim to convince the children of the usefulness of a mathematical concept or procedure and to find their assent. In this way, they adapt their thinking as to how to reason and act mathematically.

A major part of these kinds of collective argumentations are embedded in conjoint activities with the typical manipulatives, embodiments and visualizations in early maths classes. A deeper understanding of these auxiliary tools can be found in Fetzer's approach integrating these objects as additional agents in the interaction (Fetzer, 2022). These auxiliaries can be seen as typical representatives of the domain specificities of collective argumentations in early mathematics classes.

To summarize, I present a sociological interactional theory of mathematics learning. Approaching the "Despina" of learning, the specificities of the subject of early mathematics appear clearly as constitutive elements of the supportive discourses of collective argumentation. Less visible, however, is the cognitive labor of the participating individuals as they become increasingly active in these discourses. Clearly, these cognitive processes could be better realized when approaching Despina from the alternative path of psychology of cognition.

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